Scientific Notation

Scientific Notation is a way of expressing numbers. It is particularly useful as a concise way of expressing very large and very small numbers.

Scientific Notation makes use of indices, so we will have a brief look at this topic first.

Indices

Ten raised to a power is ten multiplied by itself that many times.

10^4 = 10× 10 × 10 × 10 = 10000

10^3 = 10× 10 × 10 = 1000

10^2 = 10× 10 = 100

10^1 = 10

So what is 10^0? Well every time we decrease the power by one, we get ten times smaller, so it follows that:

10^0 = 1

And if we continue

10^{-1} = $\frac{1}{10}$ = 0.1

10^{-2} = $\frac{1}{100}$ = 0.01

10^{-3} = $\frac{1}{1000}$ = 0.001

Numbers in Scientific Notation

A number in Scientific Notation looks like this:

$$2.3846 \times 10^6$$

Notice that there is one digit before the decimal place, and there can be any number of digits after. The power says we must multiply by 10 six times, i.e. move the decimal place to the right six times. So another way of writing this would be 2384600.

Whenever we multiply a number by ten, we move the decimal place once to the right
Whenever we divide by ten, we move the decimal place once to the left.

You will also see numbers in Scientific Notation with negative powers.

$$3.12 \times 10^{-4}$$

The negative power says we must divide by 10 four times, i.e. move the decimal place to the left four times. So another way of writing this would be 0.000312.

A positive power means we are dealing with a big number, and a negative power means we are dealing with a small number.

How to Express a Number in Scientific Notation

Any number can be expressed in scientific notation: 1. Move the decimal place from its starting point to be immediately after the first non-zero digit. 2. Count how many places you move – this will be the power. 3. If you move the decimal place to the left, the power will be positive. If you move the decimal place to the right, the power will be negative. 4. Check your answer. You should have a positive power for a big number, and a negative power for a small number.

Example

Express 2600000000 in Scientific Notation.

Solution

As it is written, there is no decimal place in this number, but it would be after the last zero.

We need to move the decimal place nine times to the left to place it after the first digit i.e. after the 2 and before the 6, so the power will be 9.

Answer: 2.6×10^{9}

Example

Express 0.00000218 in Scientific Notation.

Solution

We need to move the decimal place six times to the right to place it after the first non-zero digit i.e. after the 2 and before the 1, so the power will be -6.

Answer: 2.18×10^{-6}

Example

Express 34.2×10^{3} in Scientific Notation.

Solution

It may look at first as if this is in Scientific Notation already, but it isn't. We should have only one digit before the decimal place and in this case, there are two. We need to move the decimal place once to the left. This would make the number ten times smaller, so we should increase the power by one to compensate.

Answer: 3.42×10^4

If you move the decimal place to the left, increase the power. If you move the decimal place to the right, decrease the power.

Multiplying Numbers in Scientific Notation

Numbers expressed in scientific notation can be easily multiplied or divided.

Example

Multiply the following numbers, giving your answer in Scientific Notation.

$$(3\times 10^3)\times(4\times 10^4)$$

Solution

$(3\times 10^3)\times(4\times 10^4) = (3\times 10\times 10\times 10)\times(4\times 10\times 10\times 10\times 10)$

We are simply multiplying a series of numbers and we can multiply them in any order. So let's write this as

$$(3\times 4)\times(10\times 10\times 10\times 10\times 10\times 10\times 10)$$

$$= \quad 12 \quad \times \quad 10^7$$

So what we have done here is multiply the 'number parts', and added the powers. Notice that our value is not in Scientific Notation because the decimal place should be after the first digit. To complete the question let's make sure that it is.

Answer: $\quad 1.2 \quad \times \quad 10^8$

To Multiply numbers in Scientific Notation: 1. multiply the 'number parts' 2. add the powers 3. make sure the answer is in Scientific Notation

These rules still apply if we are dealing with negative powers.

Example

Simplify the following, giving your answer in Scientific Notation.

$$(3\times 10^2)\times(2\times 10^{-4})$$

Solution

$$(3\times 10^2)\times(2\times 10^{-4}) \quad = \quad (3\times 2)\times 10^{2+(-4)}$$

$$= \quad 6\times 10^{-2}$$

Dividing Numbers in Scientific Notation

To Divide numbers in Scientific Notation:

1. divide the 'number parts'
2. subtract the powers
3. make sure the answer is in Scientific Notation

Example

Simplify the following, giving your answer in Scientific Notation.

$$\frac{6\times10^{5}}{2\times10^{3}}$$

Solution

$$\frac{6\times10^{5}}{2\times10^{3}} = (6\div 2)\times 10^{5-3}$$
$$= 3\times 10^{2}$$

Example

Simplify the following, giving your answer in Scientific Notation.

$$\frac{6\times 10^{-5}}{2\times 10^{-3}}$$

Solution

$$\frac{6\times10^{-5}}{2\times10^{-3}} = (6\div 2)\times 10^{-5-(-3)}$$
$$= 3\times 10^{-5+3}$$
$$= 3\times 10^{-2}$$

Exercises

1) Express the following in scientific notation:
   a) 573000 b) 0.0000000365 c) 10
   d) 1 e) 8873 f) 0.00054

2) Evaluate the following and express the answers in scientific notation:
   a) $(3\times10^{5})\times(2\times10^{1})$ b) $(6\times10^{2})\times(2\times10^{3})$ c) $(4\times10^{8})\times(4\times10^{32})$
   d) $(8\times10^{-2})\times(5\times10^{3})$ e) $(1.2\times10^{2})\times(3.4\times10^{-3})$ f) $(1\times10^{-2})\times(2.5\times10^{-7})$
   g) $\frac{8\times10^{4}}{4\times10^{2}}$ h) $\frac{3.6\times10^{5}}{6\times10^{-3}}$ i) $\frac{6\times10^{-5}}{1.2\times10^{-3}}$

Answers

1)
   a) 5.73×10^{5} b) 3.65×10^{-8} c) 1×10^{1}
   d) 1×10^{0} e) 8.873×10^{3} f) 5.4×10^{-4}

2)
   a) 6×10^{6} b) 1.2×10^{6} c) 8×10^{40}
   d) 4×10^{2} e) 4.08×10^{-1} f) 2.5×10^{-9}
   g) 2×10^{2} h) 6×10^{7} i) 5×10^{-2}