MATHS Xtra
ALGEBRA

Unit 9

Solving quadratic equations and use of the formula

Objectives

On completion of this unit you should be able to:

1. Solve quadratic equations both graphically and algebraically.

2. Use of the formula to solve quadratic equations.

Cook, M. and Rimmer, H. (1993),
Stafford College, Earl Street, Stafford, ST16 2QR

Factorising and solving more quadratic equations

Study this example.

Example 1

Solve, $2x^2 + 5x - 12 = 0$.

Because there is a 2 in front of x^2, we can write our brackets like this,

$$(2x + ?)(x + ?) = 0$$

so that if the brackets were multiplied out, $2x \times x = 2x^2$.

We now need to find two numbers which multiply together to give -12.

The factors of -12 are,

1 x -12	3 x -4	2 x -6
-1 x 12	-3 x 4	-2 x 6.

We can try each of these pairs until we find a pair which gives an expression equal to the right hand side (R.H.S.) of the equation.

Try 1 and -12.

$$(2x + 1)(x - 12) = 2x^2 - 24x + x - 12$$

This is obviously wrong. We could try each pair in turn until we find the pair which give the original expression. The pair we need are, -3 and 4.

$$(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12$$
$$(2x - 3)(x + 4) = 2x^2 + 5x - 12$$

We can now solve our equation.

$$(2x - 3)(x + 4) = 0$$

Either $(2x - 3) = 0$

$$2x = 3$$
$$x = 1.5$$

or, $(x + 4) = 0$

$$x = -4$$

x is equal to -4 and/or 1.5.

Try the exercise on the next page.

Exercise A

Solve the following quadratic equations.

1. $2x^2 - 5x + 3 = 0$

2. $2x^2 + 13x - 7 = 0$

3. $2x^2 + x - 10 = 0$

4. $3x^2 + 4x + 1 = 0$

5. $3x^2 - x - 4 = 0$

6. $3x^2 - 11x + 6 = 0$

7. $4x^2 - 4x - 15 = 0$

8. $5x^2 + 12x - 9 = 0$

9. $-2x^2 - 5x + 18 = 0$

10. $6x^2 - 13x + 6 = 0$

Check your answers with those at the end of the unit.

Solving more quadratic equations graphically

It is possible to find the values of turning points or solve more than one quadratic equation using the same graph, provided that the equations are related to the graph.

Look at these next examples.

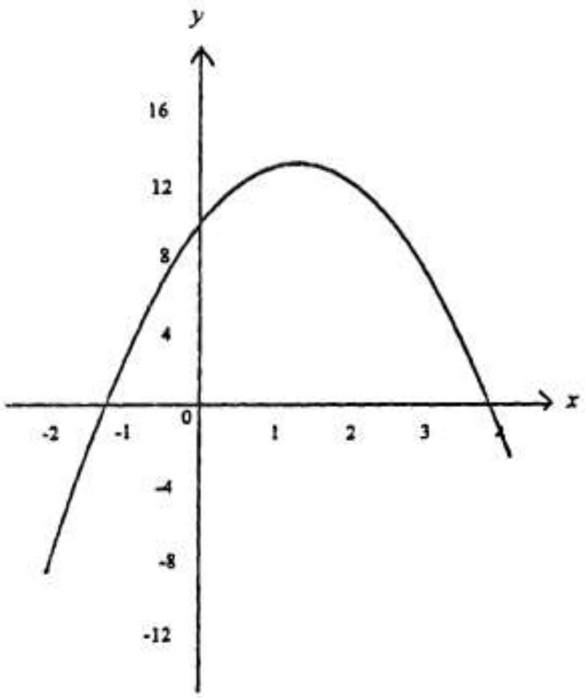
Example 2

Draw the graph of $y = -2x^2 + 5x + 10$ for values of x from -2 to 4. Calculate the turning point and state whether it is a maximum or minimum.

A table of values is needed.

Notice that x^2 is positive for all values of x, so $-2x^2$ is always negative. The whole row of numbers for values of $-2x^2$ will have negative values in it.

x	-2	-1	0	1	2	3	4
$-2x^2$	-8	-2	0	-2	-8	-18	-32
$+5x$	-10	-5	0	5	10	15	20
$+10$	10	10	10	10	10	10	10
y	-8	3	10	13	12	7	-2



When these points are plotted it can be seen that the maximum point of the graph is between $x = 1$ and $x = 2$ so another value is calculated when $x = 1.5$.

$y = -2x^2 + 5x + 10$

$y = -2(1.5)^2 + 5(1.5) + 10 = 13$

When this point is plotted it can be seen from the symmetry of the graph that the turning point occurs when $x = 1.25$ and so this value is also calculated.

$y = -2x^2 + 5x + 10$

$y = -2(1.25)^2 + 5(1.25) + 10$

$= 13.125$

The turning point is a maximum at (1.25, 13.125).

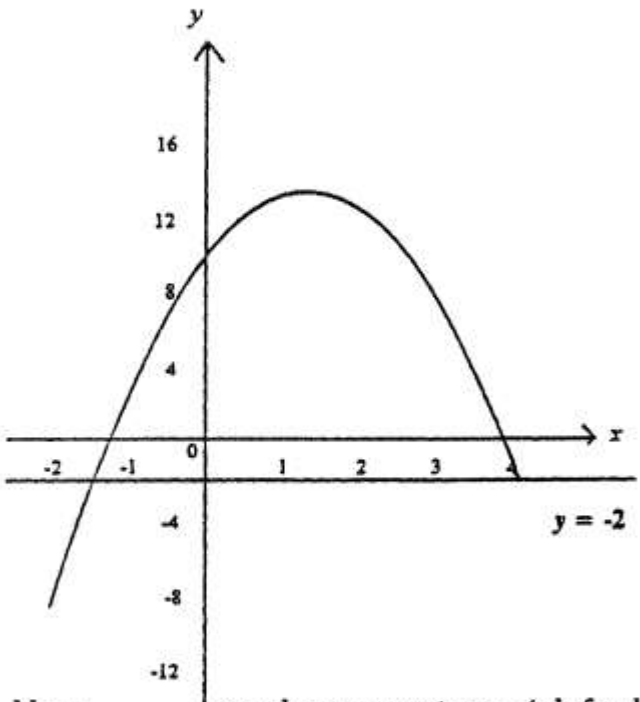
Example 3

Draw the graph of $y = -2x^2 + 5x + 10$ for values of x from -2 to 4.
From the graph, solve the equations,

a) $-2x^2 + 5x + 10 = 0$,

b) $-2x^2 + 5x + 12 = 0$.

This is the graph we produced in Example 2. It is drawn here again so that we can use the graph to solve the equations.



a) The solutions of the equation,

$$-2x^2 + 5x + 10 = 0$$

are found when the y value is 0. This is where the curve crosses the x axis.
The solutions are $x = -1.31$ and 3.81.

Now try these questions. Ask for help if you experience any

b) To solve the equation,

$$-2x^2 + 5x + 12 = 0$$

from the graph the equation must be rearranged so that the left hand side is the same as the equation of the graph.

$$-2x^2 + 5x + 12 = 0$$

The 12 can be written as 10 + 2.

$$-2x^2 + 5x + 10 + 2 = 0$$

Now subtract 2 from both sides of the equation.

$$-2x^2 + 5x + 10 = -2$$

The equation can be solved by finding the values of x when $y = -2$.
A line $y = -2$ has been drawn on the graph and the x values have been found.
From the graph, when $y = -2$, $x = -1.5$ and $x = 4$.

problems.

Exercise B

1. Draw the graph of $y = x^2 + x - 2$ for values of x between -4 and 3.
   Use your graph to solve the following equations.
   a) $x^2 + x - 2 = 0$,
   b) $x^2 + x - 5 = 0$.

2. Draw the graph of $y = -2x^2 + x + 1$ taking values of x from -3 to 3.
   Use your graph to solve the following equations.
   a) $-2x^2 + x + 1 = 0$,
   b) $-2x^2 + x + 8 = 0$.

Check your answers with those at the end of the booklet.

Formula for solving quadratic equations

There are many quadratic equations which cannot be factorised. These equations can be solved by using a formula. The general form of a quadratic equation is,

$$\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c} = 0$$

where **a** represents the number in front of the x^2 term.
Note that **a** cannot be zero because the equation would not then be a quadratic equation.
b represents the number in front of the x term.
c is the constant term.
Either **b** or **c** may be zero.

Using this form the formula used to solve the equation is,

$$x = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

Note that the whole of the numerator is divided by 2**a**.

Example 4

Solve the equation, $2x^2 + 8x - 1 = 0$ giving your answers to 2 decimal places.

Comparing this equation with the general form we see that,
a = 2, **b** = 8, **c** = -1.
Substituting in the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 + 8}}{4}$$

$$x = \frac{-8 \pm \sqrt{72}}{4}$$

Either, $x = \dfrac{-8 + 8.4852814}{4}$ and/or, $x = \dfrac{-8 - 8.4852814}{4}$

x is equal to 0.12 and/or -4.12 to two decimal places.

Example 5

Solve the equation $5x^2 - x - 2 = 0$. Answer to 2 decimal places.

a = 5, b = -1, c = -2.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{1 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{41}}{10}$$

Either,

$$x = \frac{1 + 6.4031242}{10}$$

$$x = 0.7403124$$

and/or,

$$x = \frac{1 - 6.4031242}{10}$$

$$x = -0.5403124$$

$x = 0.74$ and/or $x = -0.54$ to 2 decimal places.

Note that if you are asked to give the roots of an equation to a certain number of decimal places or significant figures, this is a very strong hint that the equation will not factorise and the formula must be used.

Try the next exercise.

Exercise C

Solve the following equations giving answers correct to two decimal places.

1. $x^2 - 3x + 1 = 0$
2. $x^2 + 6x - 5 = 0$
3. $x^2 + 7x + 4 = 0$
4. $x^2 - 2x - 7 = 0$
5. $3x^2 - 5x + 1 = 0$
6. $-2x^2 + 8x + 11 = 0$
7. $-x^2 - 9x + 1.5 = 0$
8. $2.5x^2 + 4x - 3.5 = 0$
9. $20x^2 + 15x + 2 = 0$
10. $1.6x^2 - 9x - 1.3 = 0$

Check your answers with those at the end of the booklet. The first two solutions are given in more detail for you.

Problems which lead to quadratic equations

Consider the following examples.

Example 6

A rectangle has length (x)m. and width $(x - 4)$m. The rectangle has an area of 45m^2. Find the value of x.

The area of a rectangle is given by the formula,

$$\mathbf{A = length \times breadth}$$

so using this information,

$$x \times (x - 4) = 45$$
$$x^2 - 4x = 45$$

We can rearrange this, $\quad x^2 - 4x - 45 = 0$

then factorise it. $\quad (x + 5)(x - 9) = 0$

From this we find that $x = -5$ or $x = 9$.

x cannot be negative because it is the length of a side, so x is equal to 9m.

Example 7

The displacement s, of an object at time t, after being thrown vertically upwards with initial velocity u, is given by the equation,

$$s = ut + \frac{1}{2}at^2$$

Find the first time, to three decimal places, at which the displacement is 2m. given that $u = 30$m/s. and $a = -10$m/s^2.

Substituting these values into the equation,

$$2 = 30t + \frac{1}{2}(-10)t^2$$

$$2 = 30t - 5t^2$$

Rearrange the formula,

$$5t^2 - 30t + 2 = 0$$

Using the formula,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{+30 \pm \sqrt{(-30)^2 - 4(5)(2)}}{10}$$

$$t = \frac{+30 \pm \sqrt{900 - 40}}{10}$$

$$t = \frac{+30 \pm \sqrt{860}}{10}$$

$$t = \frac{+30 \pm 29.325757}{10}$$

Either, $t = \dfrac{+30 - 29.325757}{10} = 0.0674243$

or, $t = \dfrac{+30 + 29.325757}{10} = 5.9325757$

The first time at which the displacement is 2m is the smaller value for t, the time required is 0.067 seconds to three decimal places.

Try this exercise. Ask your tutor for help if necessary.

Exercise D

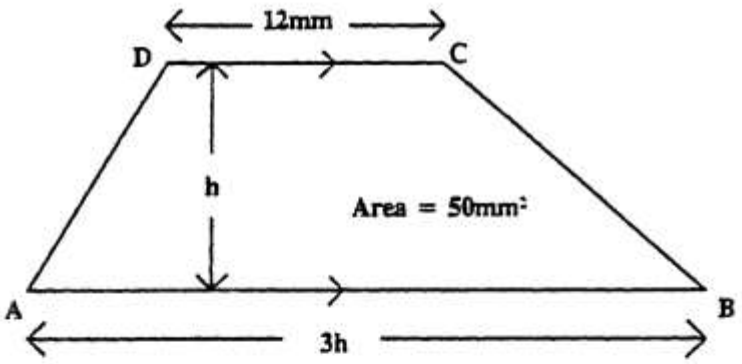
1. A triangle has a base of length $x$cm., and a perpendicular height of $(x + 3)$cm. The area of the triangle is 2cm^2. Find the value of x if the area of a triangle is, $\dfrac{\text{base x height}}{2}$.

2. The total surface area of a cylinder is given by the formula,

   $$\text{Surface area} = 2\pi r^2 + 2\pi rh$$

   where r is the radius and h is the height.
   Find the radius of a cylinder, correct to two decimal places, if the surface area is 12cm^2. and the height is 4cm.

3. The figure below shows a trapezium, where the base AB is three times the height, h. The side CD is 12mm. and the area of the shape is 50mm^2. Find the value of the height, h, correct to the nearest whole number.
   Area of a trapezium = ½ x (sum of the parallel sides) x height.



4. The total surface area of a cone is made up of two parts, the circular base and the curved surface, so that the formula is,

   $$\text{Surface area} = \pi r l + \pi r^2$$

   where r is the radius and l is the slant height. Use the formula to find the radius of a cone if the total surface area is 37cm^2. and the slant height is 5cm. Give your answer correct to two decimal places.

Check your answers with those given at the end of this unit.

Answers

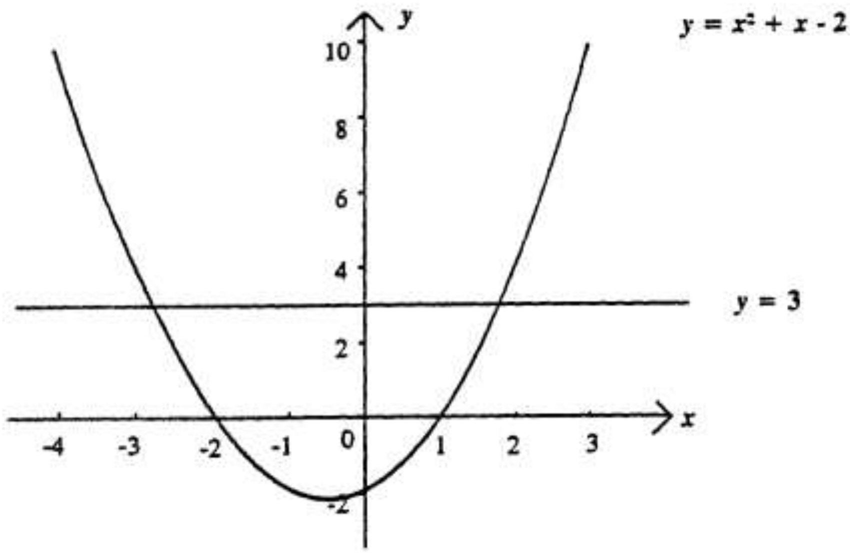
Exercise A

1. $x = \frac{3}{2}$ and/or $x = 1$
2. $x = \frac{1}{2}$ and/or $x = -7$
3. $x = -\frac{5}{2}$ and/or $x = 2$
4. $x = -\frac{1}{3}$ and/or $x = -1$
5. $x = \frac{4}{3}$ and/or $x = -1$
6. $x = \frac{2}{3}$ and/or $x = 3$
7. $x = -\frac{3}{2}$ and/or $x = \frac{5}{2}$
8. $x = \frac{3}{5}$ and/or $x = -3$
9. $x = -\frac{9}{2}$ and/or $x = 2$
10. $x = \frac{3}{2}$ and/or $x = \frac{2}{3}$

Exercise B

1.

x	-4	-3	-2	-1	0	1	2	3	-0.5
x^2	16	9	4	1	0	1	4	9	0.25
x	-4	-3	-2	-1	0	1	2	3	-0.5
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	10	4	0	-2	-2	0	4	10	-2.25



a) $x = -2$ or 1
b) $x = 1.8$ or -2.8

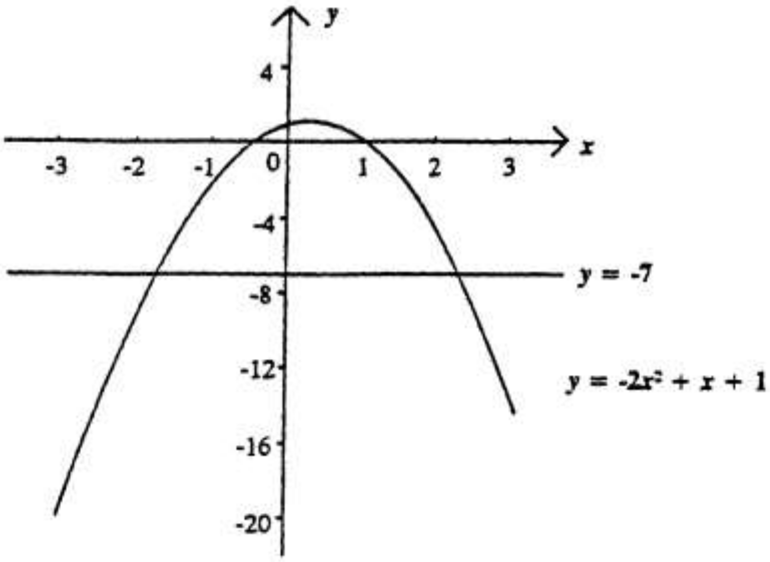
The answers for Exercise B continue on the next page.

Answers

Exercise B (*Continued*)

2.

x	-3	-2	-1	0	1	2	3	-0.5	0.5	0.25
$-2x^2$	-18	-8	-2	0	-2	-8	-18	-0.5	-0.5	-0.125
x	-3	-2	-1	0	1	2	3	-0.5	0.5	0.25
+1	1	1	1	1	1	1	1	1	1	1
y	-20	-9	-2	1	0	-5	-14	0	1	1.125



a) $x = -0.5$ or 1

b) $x = -1.8$ or 2.3

Exercise C

1. $a = 1$, $b = -3$, $c = 1$.

   $x = \frac{3 \pm \sqrt{9 - 4}}{2}$ $\quad x = \frac{3 - 2.236068}{2}$ and/or $x = \frac{3 + 2.236068}{2}$

   $x = 0.38$ and/or $x = 2.62$

2. $a = 1$, $b = 6$, $c = -5$.

   $x = \frac{-6 \pm \sqrt{36 + 20}}{2}$ $\quad x = \frac{-6 - 7.4833148}{2}$ and/or $x = \frac{-6 + 7.4833148}{2}$

   $x = -6.74$ and/or $x = 0.74$

3. $x = -6.37$ and/or $x = -0.63$
4. $x = -1.83$ and/or $x = 3.83$
5. $x = 0.23$ and/or $x = 1.43$
6. $x = -1.08$ and/or $x = 5.08$
7. $x = -9.16$ and/or $x = 0.16$
8. $x = -2.23$ and/or $x = 0.63$
9. $x = -0.58$ and/or $x = -0.17$
10. $x = -0.14$ and/or $x = 5.77$

Answers

Exercise D

1. $x = -4$ and/or $x = 1$
   A length cannot be negative and so $x = 1$cm.

2. $r = -4.4310202$ and/or $r = 0.4310202$
   A radius cannot be negative and so the radius = 0.43cm. correct to two decimal places.

3. $h = -8.1101009$ and/or $h = 4.1101009$
   A height cannot be negative and so the height = 4mm. to the nearest whole number.

4. $r = -6.7458763$ and/or $r = 1.7458763$
   A radius cannot be negative and so the radius = 1.75cm. correct to two decimal places.