MATHS Xtra
ALGEBRA

Unit 8

Difference of two squares and solution of quadratic equations

Objectives

On completion of this unit you should be able to:

1. Factorise and solve equations involving the difference of two squares.
2. Solve simple quadratic equations both algebraically and graphically.

Cook, M. and Rimmer, H. (1993),
Stafford College, Earl Street, Stafford, ST16 2QR

Difference of two squares

We have already multiplied brackets out and factorised expressions into brackets in an earlier unit. We shall now look at a special case.
Consider this example.

Example 1
Expand the expression $(x + 3)(x - 3)$ by multiplying out the brackets.

$(x + 3)(x - 3)$

$x^2 - 3x + 3x - 9$

$x \times x = x^2$
$x \times -3 = -3x$
$3 \times x = 3x$
$3 \times -3 = -9$

This will simplify to,

$x^2 - 9$

x is squared, and 9 is the square of 3.
This is referred to as the difference of two squares.

Try this short exercise.

Exercise A
Expand the following expressions by multiplying out the brackets.

1. $(x + 1)(x - 1)$
2. $(x + 2)(x - 2)$
3. $(x - 4)(x + 4)$
4. $(x + 5)(x - 5)$
5. $(x - 6)(x + 6)$
6. $(x - 7)(x + 7)$
7. $(x + 8)(x - 8)$
8. $(x + 9)(x - 9)$

Check your answers with those at the end of the unit.

Now that we have multiplied out an expression, we can reverse this process and factorise the difference of two squares.

Study the example on the next page.

Example 2
Factorise, $x^2 - 121$.

Notice that 121 is a perfect square. $11 \times 11 = 121$.
We can now reverse the process we used in the last exercise and factorise the expression.

$$x^2 - 121 = (x + 11)(x - 11).$$

Try this exercise.

Exercise B
Factorise the following expressions.
1. $x^2 - 100$
2. $x^2 - 25$
3. $x^2 - 36$
4. $x^2 - 49$
5. $x^2 - 16$
6. $x^2 - 4$
7. $x^2 - 9$
8. $x^2 - 81$

Check your answers with those at the end of the unit.

Solving an equation involving the difference of two squares

Consider this example.

Example 3
Solve $x^2 - 9 = 0$.

We factorise the left hand side of the equation as before.

$$(x + 3)(x - 3) = 0$$

The two brackets multiply together to give zero. This means that one or both of the brackets equal zero. We can say,

either, $(x + 3) = 0$	and/or, $(x - 3) = 0$
$x = -3$	$x = 3$

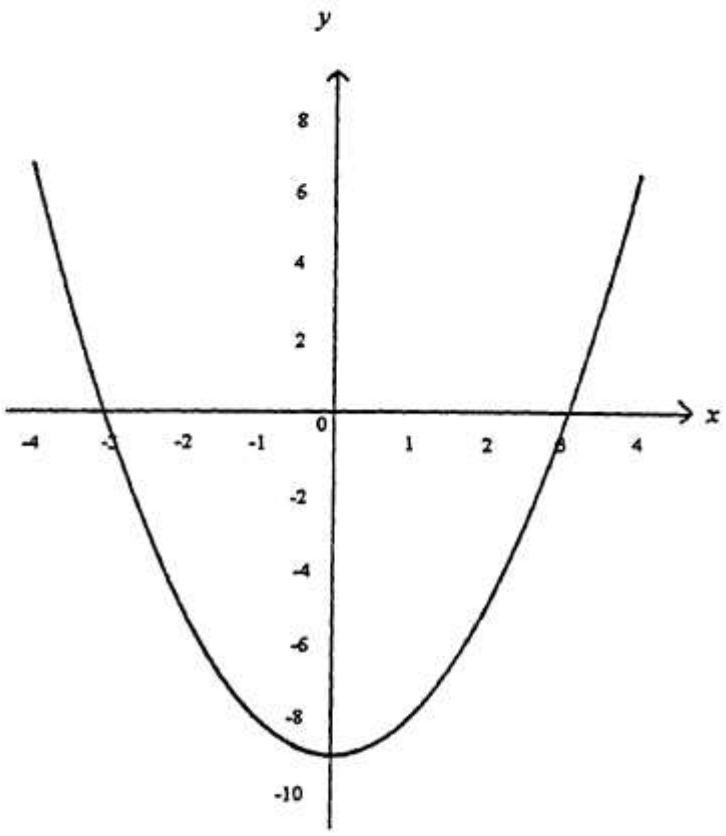
Our solutions are $x = -3$ and $x = 3$.

In Example 3, we solved the equation algebraically. We can also do this graphically.
We shall solve the same equation graphically in Example 4.

Example 4
By drawing the graph of $y = x^2 - 9$,
solve $x^2 - 9 = 0$ graphically.

We shall need a table of values.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
y	7	0	-5	-8	-9	-8	-5	0	7



The points are plotted and the graph is drawn as shown.
We now need to find the values of x to solve our equation, $x^2 - 9 = 0$.

y is equal to zero when the graph crosses the x axis. You can see that this occurs when $x = -3$ and $x = 3$.
We have obtained the same solutions as we did in Example 3.

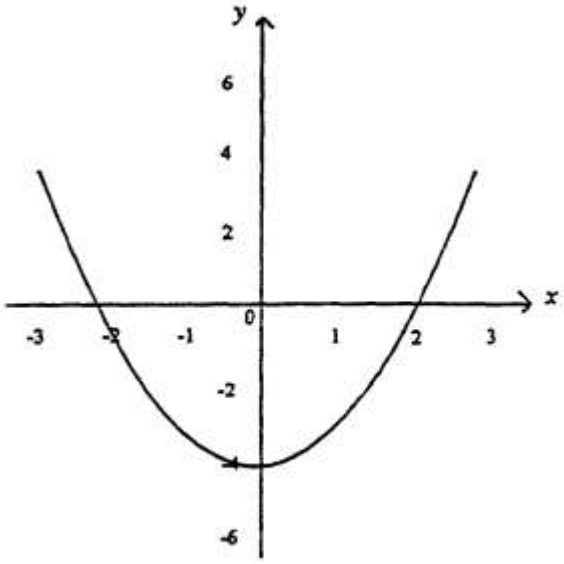
Try this exercise.

Exercise C

The following equations involve the difference of two squares, solve the equations algebraically.

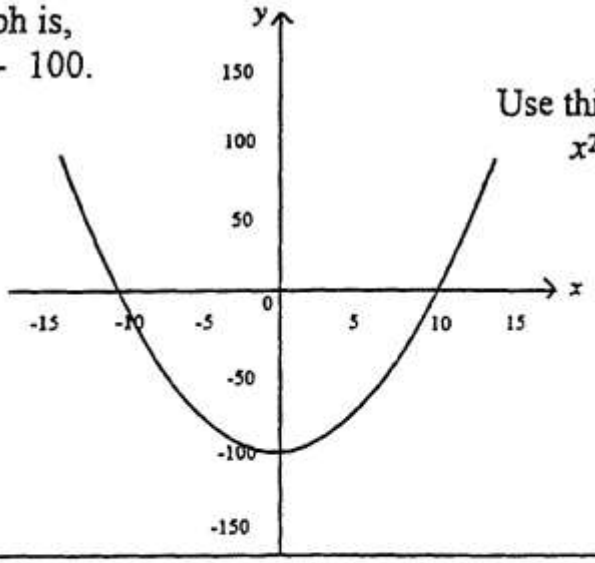
1. $x^2 - 64 = 0$
2. $x^2 - 81 = 0$
3. $x^2 - 121 = 0$
4. $x^2 - 144 = 0$
5. $x^2 - 400 = 0$

6. The graph shown below shows the equation for $y = x^2 - 4$. Use it to solve the equation, $x^2 - 4 = 0$.



7. This graph is, $y = x^2 - 100$.

Use this graph to solve, $x^2 - 100 = 0$.



Check your answers with those at the end of the unit.

Solving quadratic equations

You have already solved quadratic equations involving the difference of two squares. We shall now continue to solve other quadratic equations. Again, this can be done algebraically or graphically.

Study this example.

Example 5

Solve the equation $x^2 - x - 12 = 0$ algebraically.

We need to factorise the equation first.

$$(x + ?)(x + ?) = 0$$

We need two numbers which multiply together to give -12 and add together to give -1.

Look at the factors of -12.

1 x -12	3 x -4	2 x -6
-12 x 1	4 x -3	-2 x 6

The pair which add together to give -1 are 3 and -4.

$$(x + ?)(x + ?) = 0$$
$$(x + 3)(x - 4) = 0$$

The two brackets multiply together to give zero.

If two numbers multiply together to give zero, then one or both of the numbers must be zero. We can say that one or both of our brackets must equal zero.

If, $(x + 3) = 0$,
$x = -3$.

If, $(x - 4) = 0$,
$x = 4$.

The solution to our equation is $x = -3$ and/or $x = 4$.

Example 6

Solve the equation $x^2 - x - 12 = 0$ graphically.

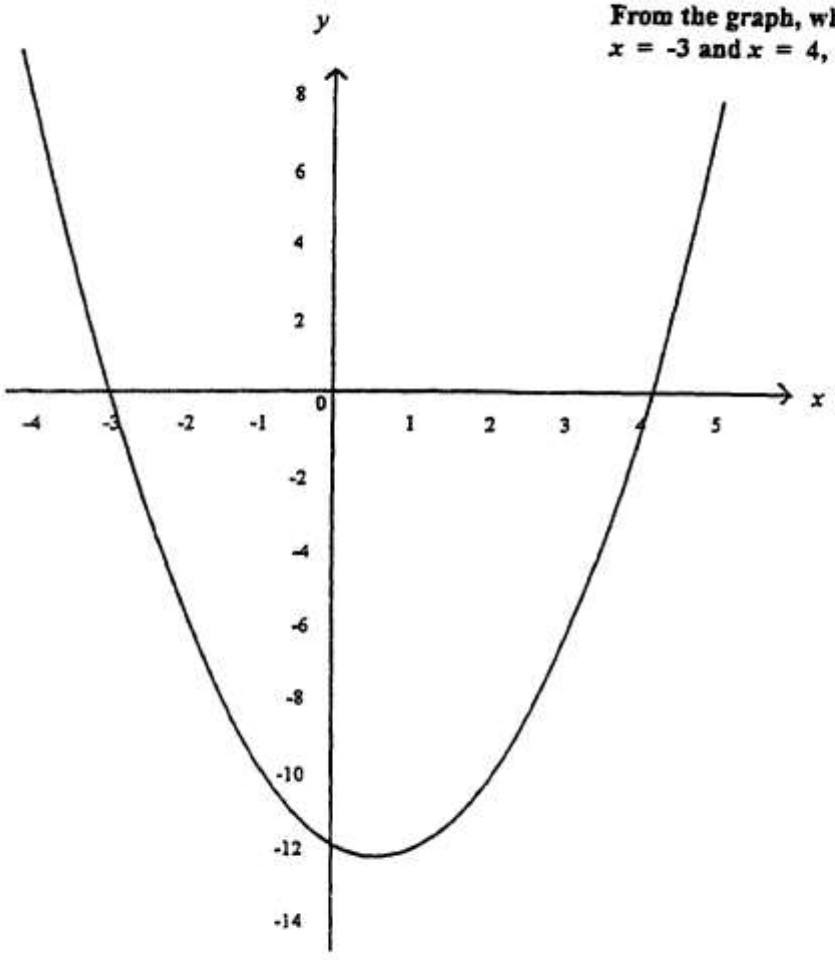
We need to draw the graph of $y = x^2 - x - 12$, then we can find the points on the graph where $y = 0$.
We shall use a table of values for x.

x	-4	-3	-2	-1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$-x$	4	3	2	1	0	-1	-2	-3	-4	-5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
y	8	0	-6	-10	-12	-12	-10	-6	0	8

The points are plotted on a graph, below. As you can see we need to find a point between $x = 0$ and $x = 1$ to find the minimum point on the graph. If we substitute $x = 0.5$, then,

$$y = (0.5)^2 - 0.5 - 12 = -12.25$$

We can plot the turning point of the graph. (0.5, -12.25).



From the graph, when $y = 0$, $x = -3$ and $x = 4$, as before.

We obtain the same answer as we did in Example 5 when we solved the same equation algebraically.

Try this exercise.

Exercise D

Solve these equations algebraically.

1. $x^2 + 5x + 6 = 0$
2. $x^2 - 6x + 8 = 0$
3. $x^2 + 3x - 10 = 0$
4. $x^2 - 4x - 12 = 0$
5. $x^2 + 2x - 3 = 0$
6. $x^2 - 11x + 18 = 0$

Solve the following equations graphically using the values suggested for x.

7. $x^2 - x - 2 = 0$ Use values of x from -3 to 4.
8. $x^2 - 5x + 4 = 0$ Use values of x from -1 to 6.
9. $x^2 - 2x - 8 = 0$ Use values of x from -3 to 5.
10. $x^2 + x - 2 = 0$ Use values of x from -3 to 3.

Check your answers with those at the end of the unit.

Answers

Exercise A

1. $x^2 - 1$
2. $x^2 - 4$
3. $x^2 - 16$
4. $x^2 - 25$
5. $x^2 - 36$
6. $x^2 - 49$
7. $x^2 - 64$
8. $x^2 - 81$

Exercise B

1. $(x + 10)(x - 10)$
2. $(x + 5)(x - 5)$
3. $(x + 6)(x - 6)$
4. $(x + 7)(x - 7)$
5. $(x + 4)(x - 4)$
6. $(x + 2)(x - 2)$
7. $(x + 3)(x - 3)$
8. $(x + 9)(x - 9)$

Exercise C

1. $x = -8$ and $x = 8$
2. $x = -9$ and $x = 9$
3. $x = -11$ and $x = 11$
4. $x = -12$ and $x = 12$
5. $x = -20$ and $x = 20$
6. $x = -2$ and $x = 2$
7. $x = -10$ and $x = 10$

Exercise D

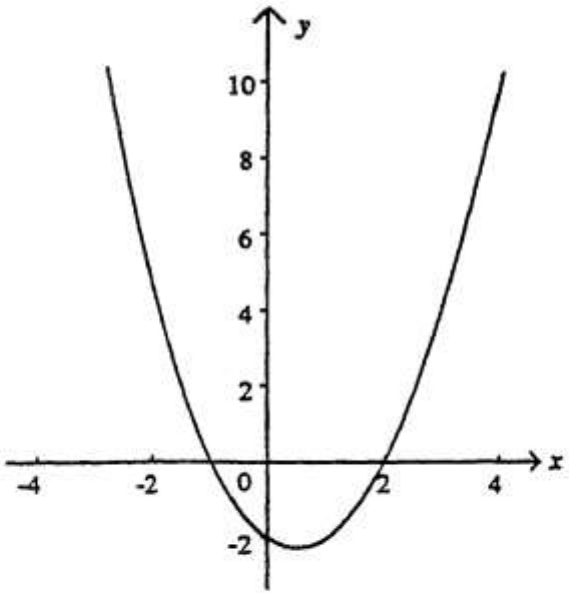
1. $x = -3$ and $x = -2$
2. $x = 2$ and $x = 4$
3. $x = -5$ and $x = 2$
4. $x = -2$ and $x = 6$
5. $x = -3$ and $x = 1$
6. $x = 2$ and $x = 9$

The answers to Exercise D are continued on the next page.

Answers Exercise D (*Continued*)

7. $y = x^2 - x - 2$

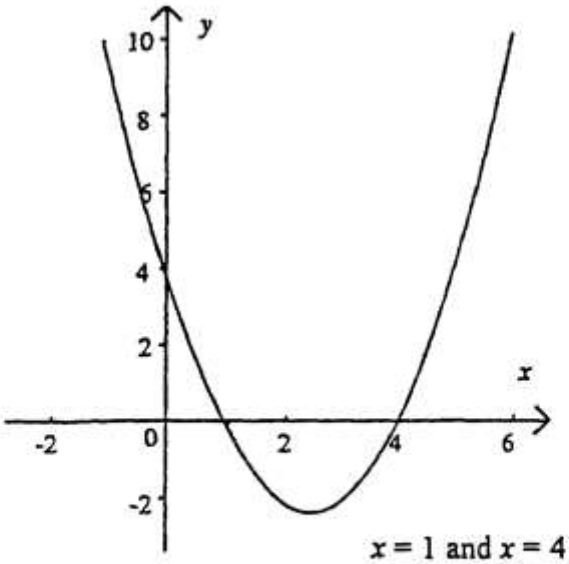
x	-3	-2	-1	0	1	2	3	4	0.5
x^2	9	4	1	0	1	4	9	16	0.25
$-x$	3	2	1	0	-1	-2	-3	-4	-0.5
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	10	4	0	-2	-2	0	4	10	-2.25



$x = -1$ and $x = 2$

8. $y = x^2 - 5x + 4$

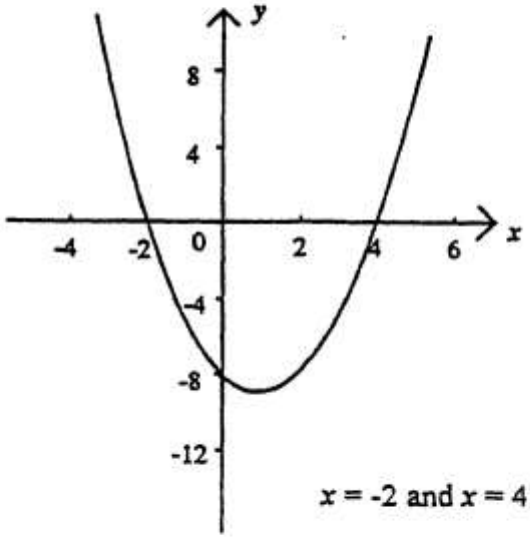
x	-1	0	1	2	3	4	5	6	2.5
x^2	1	0	1	4	9	16	25	36	6.25
$-5x$	5	0	-5	-10	-15	-20	-25	-30	-12.5
4	4	4	4	4	4	4	4	4	4
y	10	4	0	-2	-2	0	4	10	-2.25



$x = 1$ and $x = 4$

9. $y = x^2 - 2x - 8$

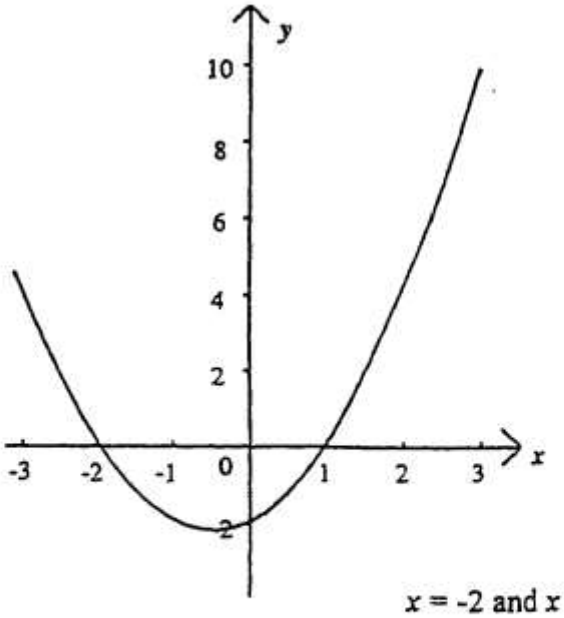
x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
$-2x$	6	4	2	0	-2	-4	-6	-8	-10
-8	-8	-8	-8	-8	-8	-8	-8	-8	-8
y	7	0	-5	-8	-9	-8	-5	0	7



$x = -2$ and $x = 4$

10. $y = x^2 + x - 2$

x	-3	-2	-1	0	1	2	3	-0.5
x^2	9	4	1	0	1	4	9	0.25
x	-3	-2	-1	0	1	2	3	-0.5
-2	-2	-2	-2	-2	-2	-2	-2	-2
y	4	0	-2	-2	0	4	10	-2.25



$x = -2$ and $x = 1$