MATHS Xtra
ALGEBRA

Unit 7

Graphs of curves

Objectives

On completion of this unit you should be able to:

1. Draw graphs of quadratic functions.

2. Use quadratic graphs in practical situations.

Cook, M. and Rimmer, H. (1993),
Stafford College, Earl Street, Stafford, ST16 2QR

Quadratic functions: Leading term positive

A function of the form,

$$y = \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$$

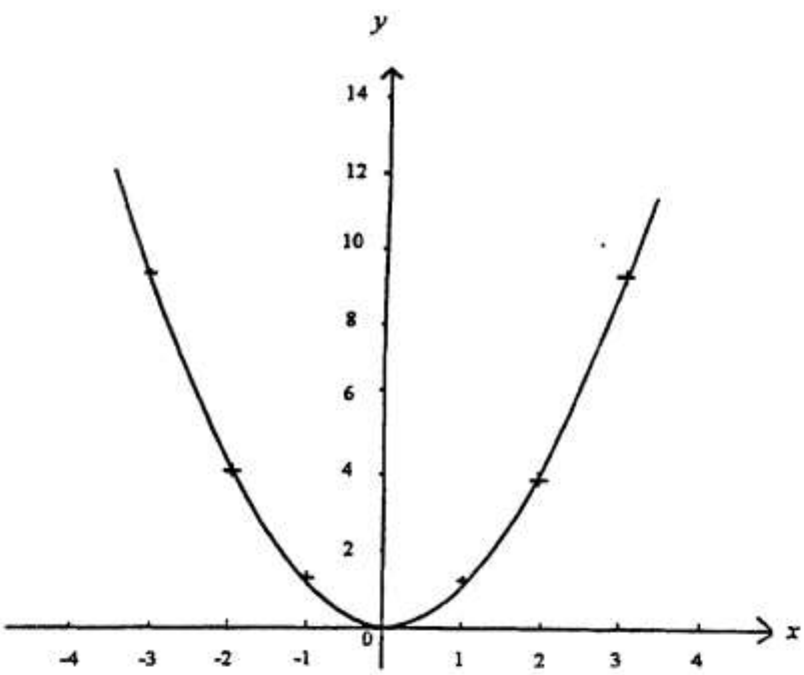
where **a** is **not** equal to 0, is a quadratic function.
Notice that there is a squared x term.
When the graphs of these functions are plotted, a smooth curve called a 'parabola' is obtained.
Study these examples.

Example 1
Draw the graph of $y = x^2$.

We can obtain a graph of this by letting x be values from -3 to +3 as follows.
We square each value of x to find the corresponding y value.

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

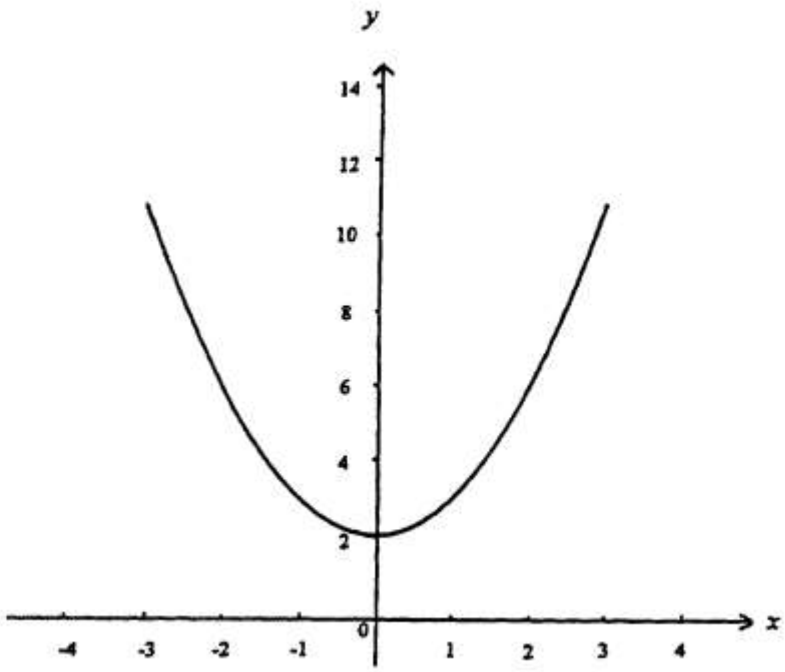
This is a graph of the function $y = x^2$.



Example 2

Using values of x from -3 to +3, form a table for the function, $y = x^2 + 2$. Sketch the graph of the function. What is the turning point of the curve?

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
x^2	9	4	1	0	1	4	9
+2	2	2	2	2	2	2	2
$y = x^2 + 2$	**11**	**6**	**3**	**2**	**3**	**6**	**11**

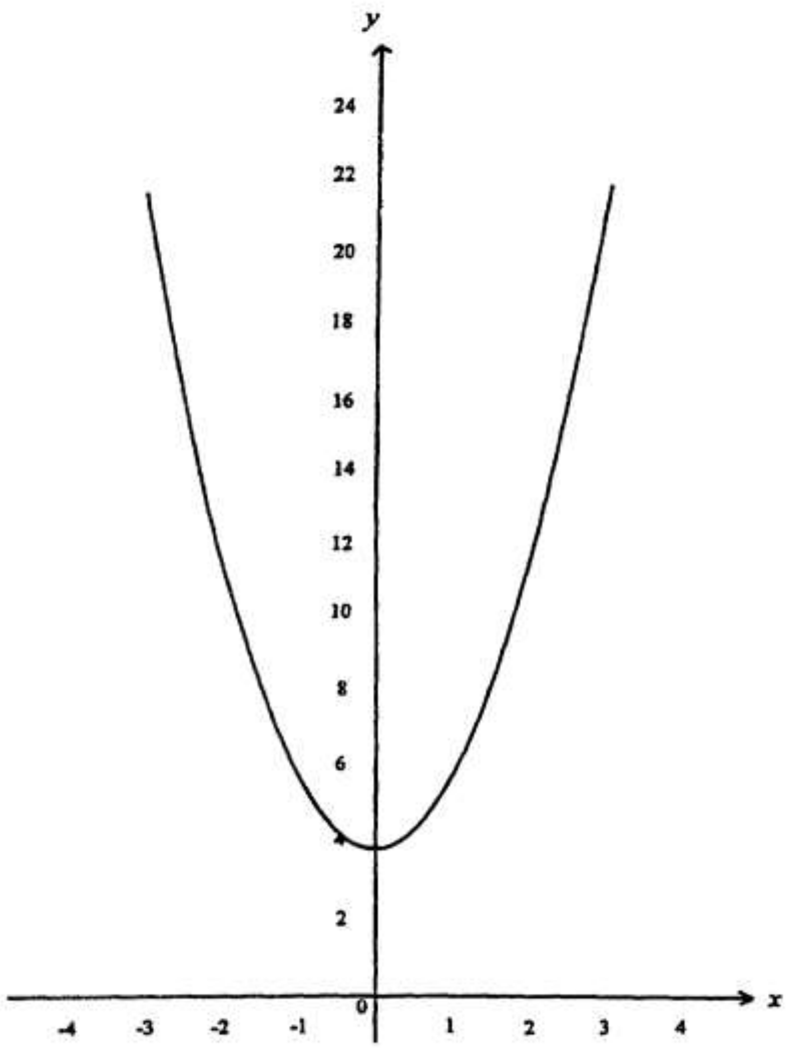


The graph turns at the point (0,2), so this is the turning point of the curve. This point has the smallest y value and is referred to as the minimum value of the curve.

Example 3

Using values of x from -3 to +3, form a table for the function $y = 2x^2 + 4$.
Sketch the graph of the function. Find the turning point of the curve.

x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
$+4$	4	4	4	4	4	4	4
$y = 2x^2 + 4$	22	12	6	4	6	12	22



The turning point of the curve is (0,4). Again it is a minimum.

The graphs obtained in the examples shown so far are all U shaped.

Try this exercise.

Exercise A
Using values of x from -3 to 3, form a table for the following functions, and sketch the graphs.
1. $y = x^2 + 1$
2. $y = 2x^2$
3. $y = x^2 - 3$
4. $y = 3x^2$

Check your answers with those at the end of the unit.

Quadratic functions: Leading term negative

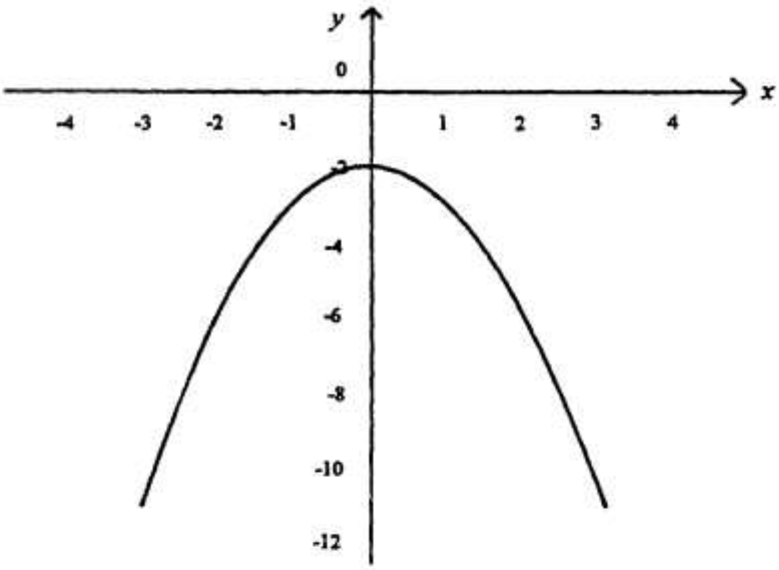
You should have noticed that all the functions sketched so far have been U shaped. Let's look again at the original general equation for a quadratic graph.

$y = \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$.

We shall now consider what happens if **a** is a negative number.

Example 4
Let **a** = -1, **b** = 0 and **c** = -2, then, $y = -x^2 - 2$.
Draw the graph, using values of x = -3 to 3 as before,

x	-3	-2	-1	0	1	2	3
$-x^2$	-9	-4	-1	0	-1	-4	-9
$y = -x^2 - 2$	-11	-6	-3	-2	-3	-6	-11



The turning point of the curve is (0,-2).
This has the highest y value and is referred to as the maximum value.

Try this exercise.

Exercise B

For each of the following functions, form a table using values of x from -3 to 3, then sketch the functions. Write the turning point of the curve in each case.

1. $y = -x^2 + 2$
2. $y = -x^2 - 4$
3. $y = -2x^2$
4. $y = -4x^2$

Check your answers with those at the end of the unit.

Note that, if the leading term of the quadratic function is negative the graph is still a parabola, but it is an **inverted** U shape.

Quadratic functions: Practical applications

Study this example.

Example 5

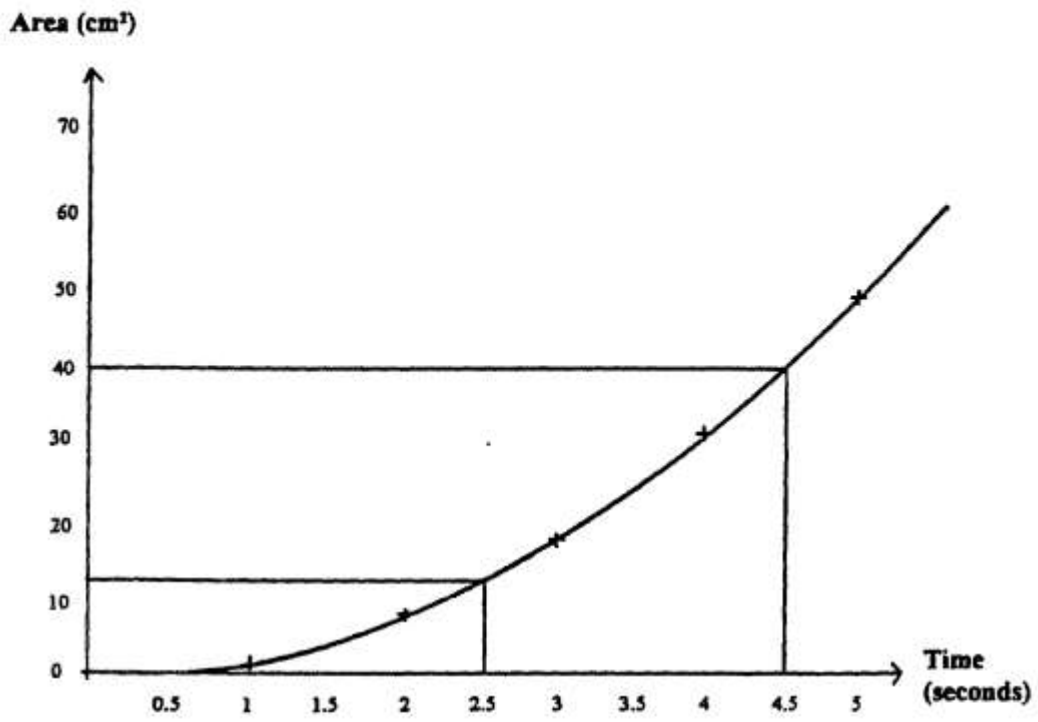
Oil is poured onto a flat surface. The area of the surface covered by the oil is measured, in cm^2. and the time taken, in seconds, is recorded. The values obtained are shown below in the table.

Area (cm^2.)	0	2	8	18	32	50
Time (s.)	0	1	2	3	4	5

Plot the points and join them with a smooth curve.
From the graph, find,

a) the time when the area covered by the oil is equal to $12.5cm^2$.,

b) the area covered by the oil after 4.5 seconds.



Using the graph,

a) when the area covered by the oil is $12.5cm^2$. the time is 2.5 seconds,

b) when the time is 4.5 seconds, the area covered by the oil is $40.5cm^2$.

Try this exercise.

Exercise C

1. A car is moving with constant acceleration. The distance covered, s, is measured in metres (m.) and the velocity, v, in metres per second (m/s.) The results obtained are shown in the table below.

Distance (m.)	0	5	12	21	32
Velocity (m/s.)	2	3	4	5	6

Plot the graph of these values. Put **Distance** on the vertical axis and **Velocity** on the horizontal axis. We say plot **Distance** against **Velocity**. From your graph find,

a) the distance moved when the velocity is 3.5m/s.,
b) the velocity when the car has moved 18m.

2. The swing of a pendulum was timed in seconds for various lengths of the pendulum measured in metres. The results were recorded in the table below.

Length, l,(m.)	0	0.57	1.01	1.58	2.28	3.10
Time (s.)	0	1.5	2.0	2.5	3.0	3.5

Plot **Length** against **Time**. This means you should put **Length** on the vertical axis and **Time** on the horizontal axis. From your graph estimate,

a) the length of the pendulum needed to have a swing whose time is 2.8 seconds,
b) the time for one swing of a pendulum if the length is 0.8m.

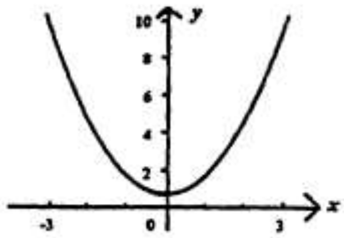
Check your answers with those at the end of the unit.

Answers

Exercise A

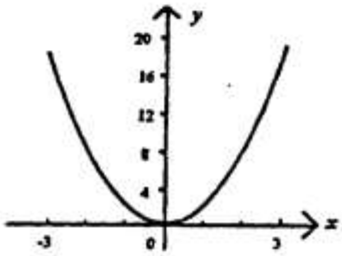
1.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
+1	1	1	1	1	1	1	1
$y=x^2+1$	10	5	2	1	2	5	10



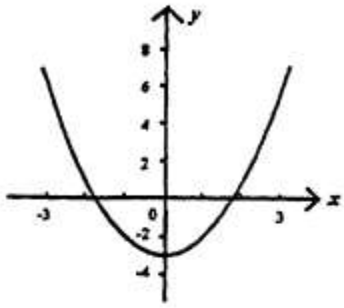
2.

x	-3	-2	-1	0	1	2	3
$y=2x^2$	18	8	2	0	2	8	18



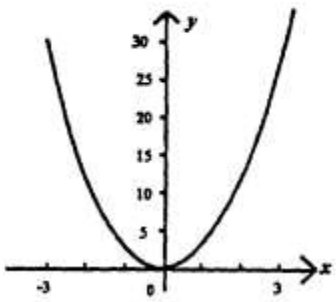
3.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
-3	-3	-3	-3	-3	-3	-3	-3
$y=x^2-3$	6	1	-2	-3	-2	1	6



4.

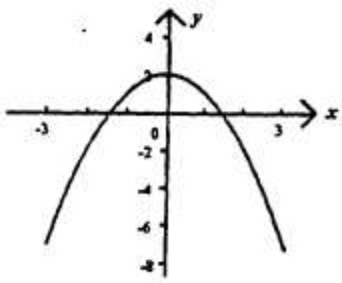
x	-3	-2	-1	0	1	2	3
$y=3x^2$	27	12	3	0	3	12	27



Exercise B

1.

x	-3	-2	-1	0	1	2	3
$-x^2$	-9	-4	-1	0	-1	-4	-9
+2	2	2	2	2	2	2	2
$y=-x^2+2$	-7	-2	1	2	1	-2	-7

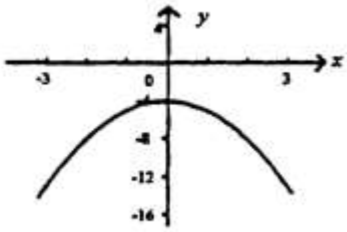


Turning point is (0,2).

Exercise B (*Continued*)

2.

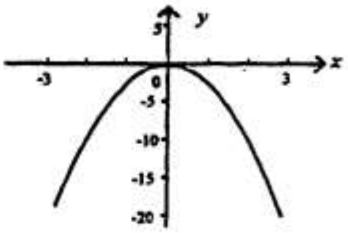
x	-3	-2	-1	0	1	2	3
$-x^2$	-9	-4	-1	0	-1	-4	-9
-4	-4	-4	-4	-4	-4	-4	-4
$y = -x^2 - 4$	-13	-8	-5	-4	-5	-8	-13



Turning point is (0,-4).

3.

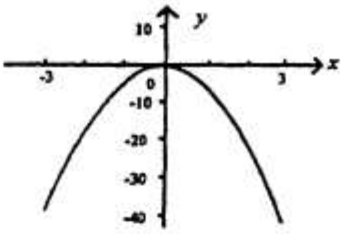
x	-3	-2	-1	0	1	2	3
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18



Turning point is (0,0).

4.

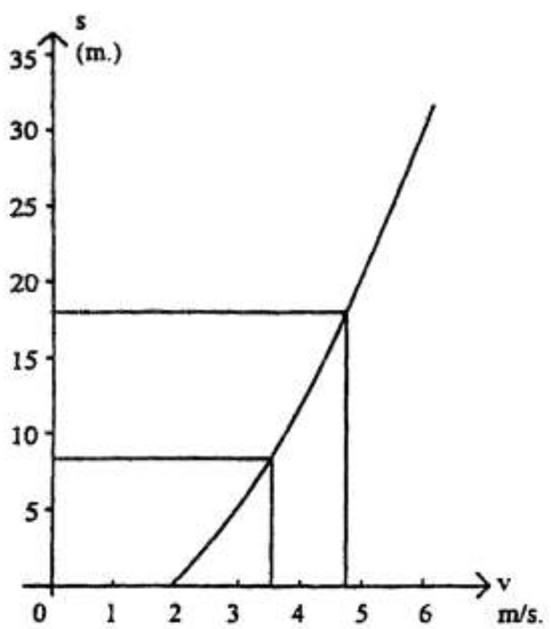
x	-3	-2	-1	0	1	2	3
$y = -4x^2$	-36	-16	-4	0	-4	-16	-36



Turning point is (0,0).

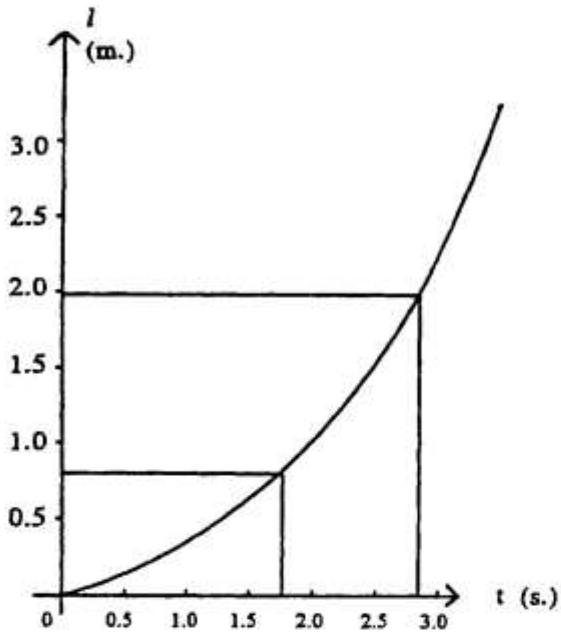
Exercise C

1.



a) S = 8.25m. b) v = 4.7m/s.

2.



a) l = 1.99m. b) t = 1.78s.