MATHS Xtra
ALGEBRA

Unit 6

Factorisation

Objectives
On completion of this unit you should understand:

1. Factorisation.

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Factorisation

This is the opposite of expanding. The aim of factorisation is to find two expressions, which, if multiplied together, give you the original expression.

Factors

3 and 4 are factors of 12
because $3 \times 4 = 12$
1, 2, 6, and 12 are also factors of 12 because

$$1 \times 12 = 12$$

and

$$2 \times 6 = 12$$

Just as we can find pairs of factors for numbers we can also do the same with some algebraic expressions. We look for common terms in the expressions given.

Study these examples

Example 1
Factorise $3x + 6$.

Both $3x$ and 6 will divide by 3. We can say that 3 is a factor of $3x$ and 6.
Divide $3x$ by 3 and we obtain x.
Divide 6 by 3 and we obtain 2.

We write the common factor outside the bracket.

$$3(x + 2)$$

To check the answer, multiply the terms inside the bracket by what is in front of the bracket. In doing this, you should arrive back at the original expression.

Example 2
Factorise $mn^3 - mn$.

Remember that this means $(m \times n \times n \times n) - (m \times n)$
m and n appear in both terms so they are common factors.
These are the common factors and will go outside the bracket.
Divide the first term by mn.

$$\frac{m \times n \times n \times n}{m \times n} = n \times n = n^2$$

Divide the second term by mn.

$$\frac{m \times n}{m \times n} = 1$$

Now we write these inside the bracket.
The answer is,

$$mn^3 - mn = mn(n^2 - 1)$$

Again check the answer by multiplying the terms inside the bracket by what is in front of the bracket.

Example 3
Factorise $4x + 8x^2 - 12x^4$.

$4x$ is the common factor.
Each term can be divided by both 4 and x.

$$4x + 8x^2 - 12x^4 = 4x(1 + 2x - 3x^3).$$

Example 4
Factorise $5x^6 + 10x^4 - 25x^3$.

$5x^3$ is the common factor.
Each term can be divided by both 5 and x^3.

$$5x^6 + 10x^4 - 25x^3 = 5x^3(x^3 + 2x - 5).$$

Now try this exercise.

Exercise A

Factorise the following.

1. $3a + 15b$
2. $3a - 21$
3. $b^3 + 4b$
4. $ab - ay$
5. $12b - 8b^2$
6. $14x^2 - 21x$
7. $2a^3 + a^2 + 5a$
8. $12a^3 - 4a^2 + 2a$
9. $6b^3 - 18b^2 + 12b$
10. $36c^3 - 9c$

Check your answers with those at the end of the booklet.

Factorising quadratics

If we expand $(2x + 3)(x - 1)$
we obtain,

$$(2x + 3)(x - 1) = 2x^2 - 2x + 3x - 3$$
$$= 2x^2 + x - 3$$

If we reverse this process and factorise,

$$2x^2 + x - 3$$

we obtain,

$$(2x + 3)(x - 1).$$

Study the examples on the next page.

Example 5
Factorise $x^2 + 5x + 4$.

We reverse the process of multiplying out the brackets.

$$(x + ?)(x + ?)$$

? x ? must equal 4.
We need two numbers which multiply together to give 4.

$1 \times 4 = 4$ and $2 \times 2 = 4$

Try 2 and 2.

$$(x + 2)(x + 2)$$

If we multiply this out we obtain,

$$x^2 + 2x + 2x + 4 = x^2 + 4x + 4$$

This is **not** the right answer.
Try 1 and 4.

$$(x + 1)(x + 4)$$

If we multiply this out we obtain,

$$x^2 + 4x + x + 4 = x^2 + 5x + 4$$

$(x + 1)(x + 4)$ is the right answer.

Example 6
Factorise $x^2 - 7x - 8$.

$$(x + ?)(x + ?)$$

? x ? must equal -8.

We need two numbers which multiply together to give -8.

$1 \times -8 = -8$ and $4 \times -2 = -8$
$-1 \times 8 = -8$ and $-4 \times 2 = -8$

We can try each pair of numbers and multiply out the brackets each time to find when we are correct. The pair we need will add together to give -7. This means we shall obtain $-7x$ as the middle term.
The two number which multiply together to give -8 and add together to give -7 are 1 and -8.
We shall use these.

$$(x + 1)(x - 8)$$

If we multiply this out we obtain,

$$x^2 - 8x + x - 8 = x^2 - 7x - 8$$

$(x + 1)(x - 8)$ is the right answer.

Example 7
Factorise $x^2 - 16$.

There is no x term in the middle. We need two numbers which multiply together to give -16 and add together to give 0.

The factors of -16 are,

1 x -16 16 x -1 8 x -2 2 x -8 4 x -4

4 and -4 add together to give 0, so we use these to factorise $x^2 - 16$.

$$(x + 4)(x - 4)$$

Multiply this out to check

$$x^2 - 4x + 4x - 16 = x^2 - 16$$
$$x^2 - 16 = (x + 4)(x - 4).$$

Try this exercise

Exercise B
Factorise the following.

1. $x^2 + 3x + 2$
2. $x^2 - 4x + 3$
3. $x^2 + 2x - 8$
4. $x^2 - x - 6$
5. $x^2 - 2x - 15$
6. $x^2 + 7x + 12$
7. $x^2 - 10x + 24$
8. $x^2 - 7x - 18$
9. $x^2 - 9$
10. $x^2 - 4$
11. $x^2 - 4x + 4$
12. $x^2 + 10x + 25$
13. $x^2 - 6x - 16$
14. $x^2 - 11x + 24$
15. $x^2 + 8x - 20$

Check your answers with those at the end of the unit.

The factorisation can be more difficult if the coefficient of x^2, that is the number in front of the term x^2 is larger than one.

Example 8
Factorise $2x^2 + 11x + 5$.

The factors of 2 are 1 and 2 so the brackets must begin with terms $1x$, which is written as simply x, and $2x$.

$$(x + ?)(2x + ?)$$

The two numbers to replace the question marks must multiply to give 5.

$$1 \times 5 = 5$$

but it is important to find which factor goes into which bracket.

$$(x + 1)(2x + 5)$$

would **not** give the correct result if multiplied out as this woud be

$$(x + 1)(2x + 5) = 2x^2 + 5x + 2x + 5$$
$$= 2x^2 + 7x + 5.$$

If instead the factors of 5 had been positioned in the brackets as follows

$$(x + 5)(2x + 1)$$

then when multiplied out the brackets give

$$(x + 5)(2x + 1) = 2x^2 + x + 10x + 5$$
$$= 2x^2 + 11x + 5.$$

$(x + 5)(2x + 1)$ is the correct answer.

Example 9
Factorise $3x^2 - 8x + 4$.

The factors of 3 are 1 and 3 and so the first terms in each bracket are x and $3x$.

$$(x - ?)(3x - ?)$$

We know the second term in each bracket must be negative and they must multiply to give 4.

$-1 \times -4 = 4$ and $-2 \times -2 = 4$

Try -1 and -4.

$$(x - 1)(3x - 4) = 3x^2 - 4x - 3x + 4 = 3x^2 - 7x + 4$$

If we change the order of the 1 and 4 we find,

$$(x - 4)(3x - 1) = 3x^2 - x - 12x + 4 = 3x^2 - 13x + 4$$

The factors of 4 which give the correct factorisation are -2 and -2.

$$(x - 2)(3x - 2) = 3x^2 - 2x - 6x + 4 = 3x^2 - 8x + 4$$

$(x - 2)(3x - 2)$ is the right answer.

If the coefficient of x^2 is not a prime number then the choice of factors becomes more difficult and when the answer is found the brackets should always be multiplied out to check if it is correct.

Example 10
Factorise $6x^2 - 7x - 3$.

The factors of 6 are,
1 x 6 or 2 x 3

The factors of -3 are,
1 x -3 or -1 x 3

The pair we need must add to give -7

1 x 6
2 x 3

1 x -3
-1 x 3

We can see that $(2 \times 1) + (3 \times -3) = -7$
So that, $6x^2 - 7x - 3$ factorises into $(2x - 3)(3x + 1)$.

Example 11
Factorise $4x^2 + 5x - 6$.

The factors of 4 are,
1 x 4 or 2 x 2

The factors of -6 are,
1 x -6 or -1 x 6
2 x -3 or -2 x 3

The pair we need will add together to give 5.

1 x 4
2 x 2

1 x -6
-1 x 6
2 x -3
-2 x 3

We can see that $(1 \times -3) + (4 \times 2) = 5$

So that $4x^2 + 5x - 6 = (x + 2)(4x - 3)$.

Try the exercise on the next page.

Exercise C

Factorise the following.

1. $2x^2 + 3x + 1$
2. $3x^2 - 5x - 2$
3. $5x^2 - 18x + 9$
4. $4x^2 + 8x - 5$
5. $6x^2 - 7x - 10$
6. $3x^2 - 14x - 5$
7. $6x^2 + 11x - 10$
8. $2x^2 + 11x + 5$
9. $6x^2 + 17x - 14$
10. $5x^2 - 44x - 9$

Check your answers with those given on the next page.

Answers

Exercise A

1. $3(a + 5b)$
2. $3(a - 7)$
3. $b(b^2 + 4)$
4. $a(b - y)$
5. $4b(3 - 2b)$
6. $7x(2x - 3)$
7. $a(2a^2 + a + 5)$
8. $2a(6a^2 - 2a + 1)$
9. $6b(b^2 - 3b + 2)$
10. $9c(4c^2 - 1)$

Exercise B

1. $(x + 2)(x + 1)$
2. $(x - 3)(x - 1)$
3. $(x + 4)(x - 2)$
4. $(x - 3)(x + 2)$
5. $(x - 5)(x + 3)$
6. $(x + 4)(x + 3)$
7. $(x - 4)(x - 6)$
8. $(x - 9)(x + 2)$
9. $(x + 3)(x - 3)$
10. $(x + 2)(x - 2)$
11. $(x - 2)(x - 2)$ or $(x - 2)^2$
12. $(x + 5)(x + 5)$ or $(x + 5)^2$
13. $(x - 8)(x + 2)$
14. $(x - 3)(x - 8)$
15. $(x - 2)(x + 10)$

Exercise C

1. $(x + 1)(2x + 1)$
2. $(x - 2)(3x + 1)$
3. $(x - 3)(5x - 3)$
4. $(2x + 5)(2x - 1)$
5. $(x - 2)(6x + 5)$
6. $(3x + 1)(x - 5)$
7. $(2x + 5)(3x - 2)$
8. $(2x + 1)(x + 5)$
9. $(2x + 7)(3x - 2)$
10. $(5x + 1)(x - 9)$