MATHS Xtra ALGEBRA

Unit 5

Simultaneous equations

Objectives

On completion of this unit you should be able to:

1. Solve linear simultaneous equations, graphically and algebraically.

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Simultaneous equations

Look at the equation,

$$y = 12 - x$$

you will see that it is not possible to find just one value of x and one value for y that will satisfy the equation.
For example we could let,

$$x = 3$$

which will give,

$$y = 12 - 3 = 9$$

or we could let,

$$x = -1$$

which will give,

$$y = 12 - (-1) = 13.$$

There are many more values of x and y that will satisfy this equation. Can you think of some more?

To make the solution of the equation unique, you need another equation involving the same variables (x and y).
For example if you use the equations,

$$y = 12 - x$$
$$y = 6 + x$$

there would be only one value of x and one value of y, that would satisfy both equations simultaneously. The values are $x = 3$ and $y = 9$.

Such equations are called **simultaneous equations** because they are solved by the same values of the variables.

Simultaneous equations can be solved by two methods.

Method 1 Graphically
Method 2 Algebraically

Study the example on the next page.

Example 1

Solve the simultaneous equations,

$$y = 12 - x$$
$$y = 6 + x$$

graphically, by plotting the following values for the two graphs.

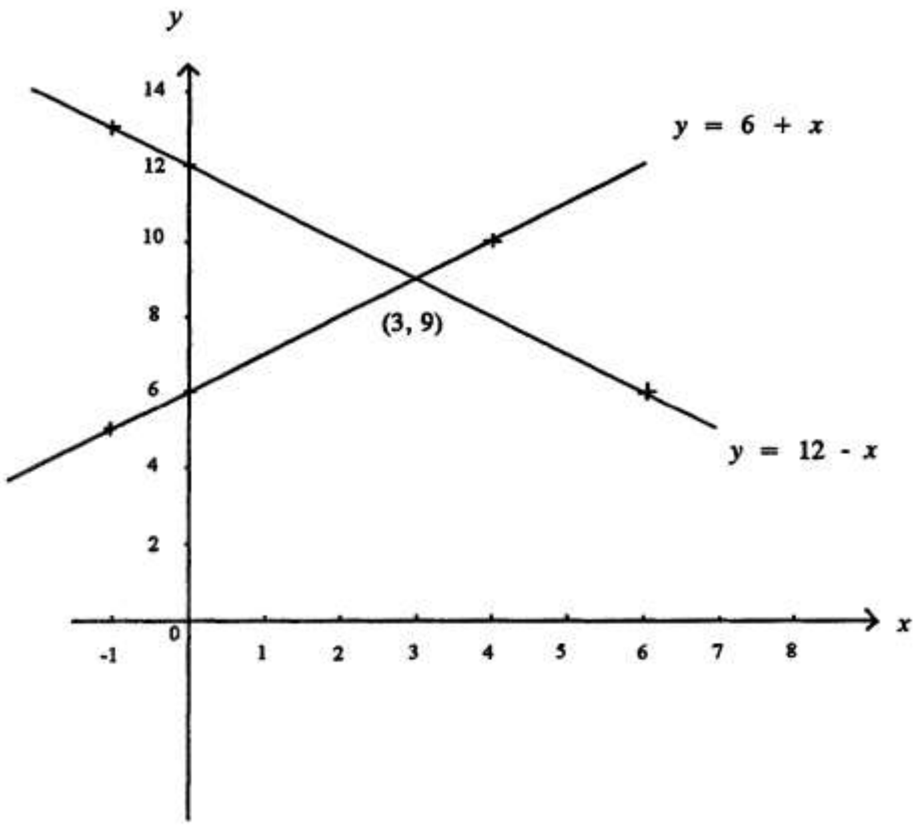
$y = 12 - x$

x	-1	0	6
$y = 12 - x$	13	12	6

$y = 6 + x$

x	-1	0	4
$y = 6 + x$	5	6	10

The graphs of each of the equations are plotted using the same scales and axes, thus superimposing one graph on top of the other, as illustrated in the diagram below.



You can see from the diagram that the graphs intersect at the point (3, 9), so the solution of the two equations is $x = 3$ and $y = 9$.
It is the only point common to both graphs.

Example 2

Solve the simultaneous equations,

$$y = 12 - x$$
$$y = 6 + x$$

algebraically.

First number the equations $\quad y = 12 - x \quad (1)$
$\quad y = 6 + x \quad (2)$

The solution to the two equations can be found when the 'y' value from equation (1) is equal to the 'y' value of equation (2).
If we replace the expression for y from equation (1) into equation (2) we get,

$$12 - x = 6 + x$$

Add x to both sides.

$$12 - x + x = 6 + x + x$$
$$12 = 6 + 2x$$

Subtract 6 from both sides.

$$12 - 6 = 6 - 6 + 2x$$
$$6 = 2x$$

Divide both sides by 2.

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

Now by substituting this value, $x = 3$ for x into either equation (1) or (2) you will find the corresponding y value.

Into (1)	or	Into (2)
$y = 12 - 3$		$y = 6 + 3$
$y = 9$		$y = 9$

So the solution is $x = 3$, $y = 9$.
Note that this is the same answer that we obtained in Example 1 when we solved the same simultaneous equations graphically.

Study this example.

Example 3

Solve the simultaneous equations,

$$y = 2x - 4 \qquad (1)$$
$$2y = x + 1 \qquad (2)$$

The solution to the two equations can be found when the 'y' value from equation (1) is equal to the 'y' value of equation (2).

If we replace the expression for y from equation (1) into equation (2) we get,

$$2(2x - 4) = x + 1$$

Multiply out the brackets.

$$4x - 8 = x + 1$$

Take x from both sides.

$$4x - 8 - x = x + 1 - x$$
$$3x - 8 = 1$$

Add 8 to both sides.

$$3x - 8 + 8 = 1 + 8$$
$$3x = 9$$

Divide both sides by 3.

$$x = 3$$

Substitute this back into either equation (1) or equation (2).

We shall substitute $x = 3$ into equation (1).

$$y = 2x - 4$$
$$y = 6 - 4$$
$$y = 2$$

The solution is, $x = 3$, $y = 2$.

Try the exercise on the next page.

Exercise A

Solve the following pairs of simultaneous equations graphically.
Suitable points are given for you to plot.

1. $y = 10 - 2x$
   $y = x + 1$

$y = 10 - 2x$			
x	-1	0	4
$y = 10 - 2x$	12	10	2

$y = x + 1$			
x	-1	0	8
$y = x + 1$	0	1	9

2. $y = 2x + 1$
   $y = 11 - 3x$

$y = 2x + 1$			
x	0	2	4
$y = 2x + 1$	1	5	9

$y = 11 - 3x$			
x	-1	0	3
$y = 11 - 3x$	14	11	2

Solve the following simultaneous equations algebraically.

3. $y = 2x + 5$
   $y = 11 - 4x$

4. $y = 4x + 7$
   $y = 2x + 5$

5. $y = x + 2$
   $y = -14 - 3x$

6. $2y = x + 10$
   $y = 3x + 5$

7. $3y = 2x - 18$
   $y = 16 - 3x$

8. $y = -x + 1$
   $2y = x + 5$

9. $2y = x - 1$
   $y = -2 - x$

10. $y = x - 3$
    $-4y = 5x - 6$

Check your answers with those shown at the end of the booklet.

Modelling equations

In a practical situation the information is often not given neatly in equation form but must be interpreted into simultaneous equations before being solved as seen at the beginning of this unit.

Example 4

Find two numbers such that their sum is 53 and their difference is 5.

Let the two numbers be x and y.

The sum of the two numbers is $x + y = 53$ (1)

The difference in the two numbers is $x - y = 5$ (2)

From (1), $x = 53 - y$ and from (2), $x = 5 + y$

Therefore $53 - y = 5 + y$

Add y to both sides, $53 = 5 + 2y$

Subtract 5 from both sides $53 - 5 = 2y$

$2y = 48$

$y = 24$

Substitute back into (1) $x + 24 = 53$

$x = 53 - 24 = 29$

The two numbers are 29 and 24.

Example 5

Two quantities are connected by the law, $y = \mathrm{m}x + \mathrm{c}$. It is found that when $x = 1, y = 7$ and when $x = 3, y = 17$. Find the values of m and c without drawing a graph.

Substitute the first set of data, $x = 1, y = 7$ into the equation $y = \mathrm{m}x + \mathrm{c}$.

$7 = \mathrm{m} + \mathrm{c}$ (1)

Substitute the second set of data, $x = 3, y = 17$ into the equation $y = \mathrm{m}x + \mathrm{c}$.

$17 = 3\mathrm{m} + \mathrm{c}$ (2)

From (1) $\mathrm{c} = 7 - \mathrm{m}$ from (2) $\mathrm{c} = 17 - 3\mathrm{m}$,

so, $7 - \mathrm{m} = 17 - 3\mathrm{m}$

Add 3m to both sides, $7 - \mathrm{m} + 3\mathrm{m} = 17$

$7 + 2\mathrm{m} = 17$

Subtract 7 from both sides, $2\mathrm{m} = 17 - 7$

$2\mathrm{m} = 10$

$\mathrm{m} = 5$

Substitute m = 5 into (1) $7 = 5 + \mathrm{c}$

$\mathrm{c} = 2.$

Consider the last example on the next page.

Example 6
Three components of type A plus five components of type B together cost 76 pence to manufacture, but five components of type A and three components of type B cost 68 pence. Find the cost to manufacture each type of component.

Let A be the cost to manufacture type A and B be the cost of type B.

Then, $3A + 5B = 76$ (1)

and, $5A + 3B = 68$ (2)

Rearrange each equation to make A the subject.

From (1) $A = \frac{76 - 5B}{3}$ (1)

From (2) $A = \frac{68 - 3B}{5}$ (2)

Therefore

$$\frac{76 - 5B}{3} = \frac{68 - 3B}{5}$$

Multiply through by 15,

$$5 \times (76 - 5B) = 3 \times (68 - 3B)$$
$$380 - 25B = 204 - 9B$$

Rearranging, $176 = 16B$

$B = 11$ pence.

Substitute in (1) $3A + 5(11) = 76$

$3A = 21$

$A = 7$ pence.

Try this last exercise and check your answers with those given at the end of the unit.

Exercise B

1. Find two numbers whose sum is 57 and whose difference is 13.

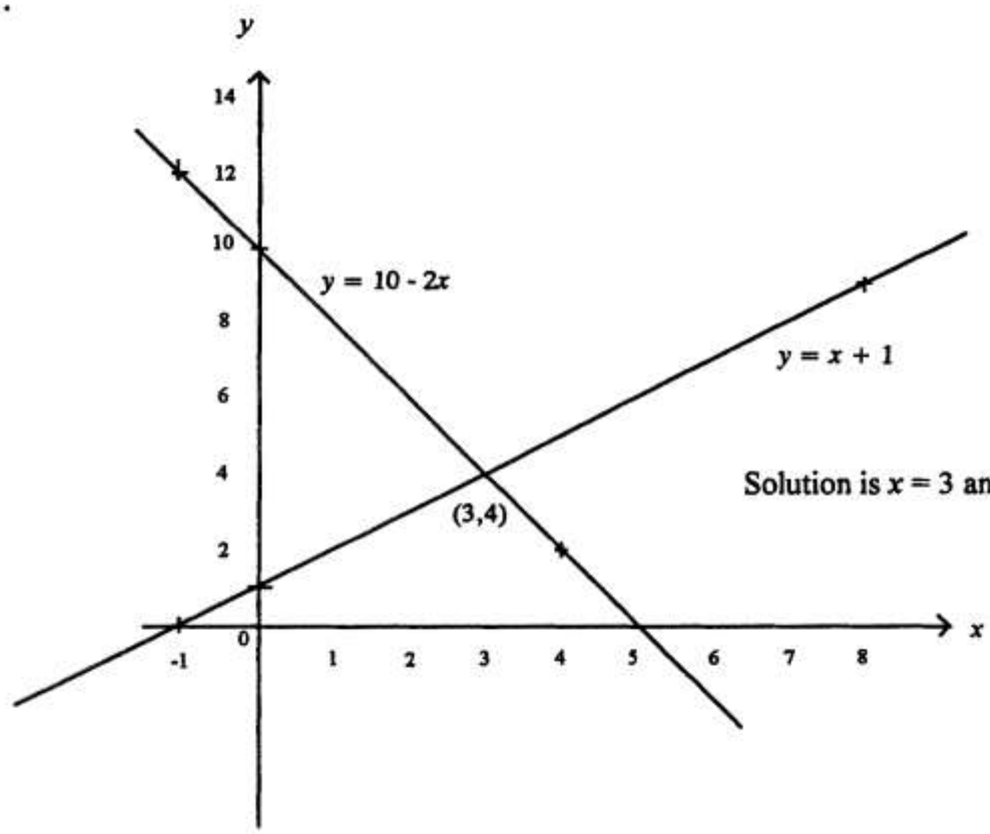
2. If S and T are believed to be connected by the law,
$$S = aT + b.$$
Find the constants a and b if S equals 16 when T equals 2 and S equals 11 when T is 1.

3. In a factory there are two rates of pay £x per hour for basic time and £y per hour for overtime. If the employees work a basic week of 40 hours, find x and y if a worker earns £370 for working 45 hours and £340 for working 42 hours.

Answers

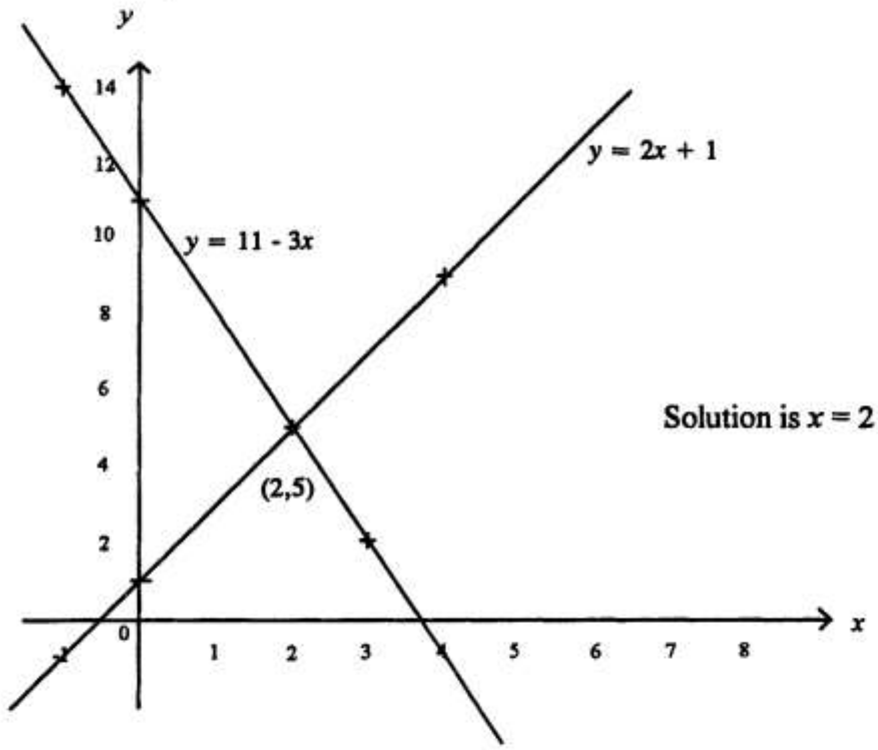
Exercise A

1.



Solution is $x = 3$ and $y = 4$

2.



Solution is $x = 2$ and $y = 5$

The answers to Exercise A are continued on the next page.

Answers

Exercise A (*Continued*)

3. $x = 1,\quad y = 7$
4. $x = -1,\quad y = 3$
5. $x = -4,\quad y = -2$
6. $x = 0,\quad y = 5$
7. $x = 6,\quad y = -2$
8. $x = -1,\quad y = 2$
9. $x = -1,\quad y = -1$
10. $x = 2,\quad y = -1$

Exercise B

1. The numbers are 22 and 35.
2. a = 5 and b = 6
3. $x = 8$ and $y = 10$