MATHS Xtra

Unit 44

Binary, octal and hexadecimal systems

Objectives

On completion of this unit you should understand:

1. Conversion from binary to denary systems and vice versa.
2. Addition, subtraction, multiplication and division in binary and octal systems
3. Hexadecimal (hex) systems.
4. Conversion between hex, octal and binary systems.
5. Two's and one's complement.

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The binary system

The common number system is called the denary system. We use the digits

Ø, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Let's consider how the denary system works.

Consider the number 2453, we can write this under our thousands, hundreds, tens and units columns using the number base 10, as follows,

1000 or 10^3	**100 or 10^2**	**10 or 10^1**	**1 or 10^0**
2	4	5	3

so we have, $2 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 = 2453.$

The binary system is a system using base 2 and only uses the digits Ø and 1. Our headings are similar to the denary system, which uses base 10, but we now use base 2 and our headings are as follows,

8 or 2^3 **4 or 2^2** **2 or 2^1** **1 or 2^0.**

Consider this example.

Example 1

Convert the binary number $1Ø1Ø1_2$ into the denary system.

We put the digits under the suitable headings. Note that the subscript 2 written next to the number tells us that the binary system, base 2, is being used.

16 or 2^4	**8 or 2^3**	**4 or 2^2**	**2 or 2^1**	**1 or 2^0.**
1	Ø	1	Ø	1

Our number in the denary system is therefore,

$16 \times 1 + 8 \times Ø + 4 \times 1 + 2 \times Ø + 1 \times 1 = 21.$

Try this exercise, then check your answers with those at the end of the unit.

Exercise A

Convert the following binary numbers to denary numbers.

The headings you should use are,

128 or 2^7 64 or 2^6 32 or 2^5 16 or 2^4 8 or 2^3 4 or 2^2 2 or 2^1 1 or 2^0.

1.	$1Ø1ØØ_2$	6.	$111ØØØ1_2$
2.	$1ØØØ1ØØ_2$	7.	$11Ø1_2$
3.	$11Ø1ØØ_2$	8.	$111ØØ1Ø_2$
4.	$1111ØØ_2$	9.	$1Ø1Ø1ØØ_2$
5.	$111Ø1ØØ_2$	10.	$11Ø11ØØ1_2$

Conversion from the denary system to the binary system

Consider this example.

Example 2

Convert the denary number 203 into a binary number.

First write the headings,

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1

Our number is smaller than 256, so we need,

1 x 128	203 - 128 = 75 so we need 1 x 64
1 x 64	75 - 64 = 11
	11 is smaller than 32 and 16 so we do not need 32 or 16
Ø x 32	
Ø x 16	we need 1 x 8
1 x 8	11 - 8 = 3
	3 is smaller than 4 so we do not need 4,
Ø x 4	we need 1 x 2
1 x 2	3 - 2 = 1 we need 1 x 1
1 x 1	

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1
	1	1	Ø	Ø	1	Ø	1	1

1 x 128 + 1 x 64 + Ø x 32 + Ø x 16 + 1 x 8 + Ø x 4 + 1 x 2 + 1 x 1 = 203

203 = $11\text{Ø}\text{Ø}1\text{Ø}11_2$.

Try this exercise.

Exercise B

Convert the following denary numbers to binary numbers.

1. 13
2. 101
3. 155
4. 27
5. 100
6. 23
7. 30
8. 32
9. 45
10. 212

Check your answers with those at the end of the unit.

Addition and subtraction of binary numbers

Consider these examples.

Example 3

Calculate,

1 1 1 Ø Ø 1 1 $Ø_2$
1 Ø 1Ø 1 1 1 $Ø_2$ +

____

In the denary system, base 10, we 'carry' one, one group of ten, when we get 10 or over.
In the binary system, we 'carry' one, one group of two, when we get 2, so that 1 + 1 = 1Ø.
Similarly 1 + 1 + 1 = 11 and 1 + 1 + 1 + 1 = 1ØØ, so we can proceed as follows,

1 1 1 Ø Ø 1 1 $Ø_2$
1 Ø 1 Ø 1 1 1 $Ø_2$ +
1 1 Ø Ø 1 Ø 1 Ø $Ø_2$
1 1 1   1 1 1

Example 4

Calculate,

1 1 1 1 Ø 1 1 $Ø_2$
1 Ø 1 1 1 1_2 -

____

Again this is the same method as the denary system, base 10, when we 'borrow' one, we 'borrow' one group of ten.
In the binary system, when we 'borrow' one, we 'borrow' one group of two, so that, considering the 2^0 column, Ø - 1 is impossible so we 'borrow one' from the next column. The 'one' we have borrowed is 2, so we say 2 - 1 = 1 and put a Ø in the 2^1 column.

~~2~~1 2 2 2
1 1 1 ~~1~~ Ø ~~1~~ ~~1~~ $Ø_2$
1 Ø 1 1 1 1_2 -
1 1 Ø Ø Ø 1 1 1_2

Now the 2^1 column, Ø - 1 is impossible, borrow one group of two as before. 2 - 1 = 1.

The process continues as for subtraction in the denary system, but each time a 2 is borrowed.

Try this exercise.

Exercise C
Calculate the folowing. Set the questions out as we did in the examples.

1. 1Ø1ØØ11Ø$_2$ + 1Ø111Ø$_2$
2. 11Ø111Ø$_2$ + 1Ø111Ø$_2$
3. 111111Ø$_2$ + 1ØØ11Ø$_2$
4. 1Ø11ØØ$_2$ + 1Ø111ØØ$_2$
5. 1Ø1ØØ11$_2$ + 1Ø111Ø$_2$
6. 11Ø1111$_2$ - 1ØØ1Ø1Ø$_2$
7. 111Ø1Ø$_2$ - 1ØØØ11Ø$_2$
8. 11Ø11Ø$_2$ - 1ØØ1$_2$
9. 1Ø11ØØ11$_2$ - 1Ø1Ø11Ø$_2$
10. 1Ø1ØØØ1$_2$ - 11Ø111$_2$

Check your answers with those at the end of the unit.

Multiplication of binary numbers

Consider this example.

Example 5
Calculate 1Ø 111$_2$ x 1Ø1$_2$.

Set this out and multiply as for denary numbers. (For ease, we shall not write the subscript 2.)

```
    1 Ø 1 1 1
          1Ø1  x
1 Ø 1 1 1 Ø Ø
  Ø Ø Ø Ø Ø Ø
    1 Ø 1 1 1
1 1 1 Ø Ø 1 1
  1 1 1
```

Remember that the addition must also be done in the binary system.

1Ø 111$_2$ x 1Ø1$_2$ = 111ØØ11$_2$.

Division of binary numbers

Work through the following example.

Example 6

Calculate $101010_2 \div 110_2$.

We set this out as for a denary long division question.

```
             1 1 1
        _____________
   110 ) 1 0 10 10
         1 10
         10 0 1
           1 1 0
             1 10
             1 10
                0
```

$101010_2 \div 110_2 = 111_2$.

Try this exercise.

Exercise D

Calculate the following. Set the questions out as shown in the examples.

1. $11101_2 \times 11_2$
2. $10011_2 \times 101_2$
3. $10111_2 \times 100_2$
4. $11110_2 \times 10_2$
5. $101010_2 \times 111_2$
6. $101010_2 \div 11_2$
7. $111111_2 \div 111_2$
8. $11010_2 \div 10_2$
9. $1100100_2 \div 101_2$
10. $10011011_2 \div 101_2$

Check your answers with those at the end of the unit.

The octal system

This number system is to base 8. We use the digits Ø 1 2 3 4 5 6 7. Our column headings are,

4096 or 8^4 512 or 8^3 64 or 8^2 8 or 8^1 1 or 8^0.

The same principles as base 10 and base 2 are employed.

Addition and subtraction

For addition, we 'carry' one group of 8, in questions involving subtraction, we 'borrow' one group of 8.

Consider this example.

Example 7

Calculate,

a) $2467_8 + 336_8$,

b) $543_8 - 126_8$.

a)
$$\begin{array}{r} 2467_8 \\ \underline{336_8} + \\ \underline{3Ø25_8} \\ 1\,1\,1 \end{array}$$

Add 6 + 7 = 13. This is one group of 8 and 5 'left over'. Write the 5 and 'carry 1', the one group of 8. 6 + 3 + 1 = 10. This is one group of 8 and 2 'left over'. Write the 2 and 'carry 1'. 4 + 3 + 1 = 8. This is just one group of 8 and nothing 'left over'. Write the zero and 'carry 1'.
Finally, 2 + 1 = 3.

b)
$$\begin{array}{r} {\scriptstyle 3\ 8}\ \ \\ 5\,\not{4}\,3_8 \\ \underline{1\,2\,6_8} - \\ \underline{4\,1\,5_8} \end{array}$$

3 - 6 is not possible, so we 'borrow' one group of 8.
8 + 3 = 11. 11 - 6 = 5.
Write the 5.
We have borrowed one from the four, so we now continue, 3 - 2 = 1. Write 1.

Try this exercise, then check your answers with those at the end of the unit.

Exercise E

Calculate the following.

1. $3367_8 + 351Ø_8$	6. $55211_8 - 3Ø72_8$
2. $5547_8 + 442_8$	7. $435Ø_8 - 573_8$
3. $4122_8 + 3Ø72_8$	8. $3117_8 - 262Ø_8$
4. $4325_8 + 1352_8$	9. $26514_8 - 4Ø71_8$
5. $33611_8 + 2252_8$	10. $55231_8 - 11323_8$

Multiplication and division in the octal system

Consider this example.

Example 8

Calculate,

a) $275_8 \times 6$,

b) $322_8 \div 5$.

a)

$$\begin{array}{r} 2\,7\,5 \\ 6\ \times \\ \hline \underline{2\,1\,5\,6} \\ {\scriptstyle 2\ 5\ 3} \end{array}$$

$5 \times 6 = 30$.
This is 3 groups of 8 and 6 left over. Write 6, carry 3.
$7 \times 6 = 42$ $\quad$ $42 + 3 = 45$.
This is 5 groups of 8 and 5 left over. Write 5, carry 5.
$2 \times 6 = 12$ $\quad$ $12 + 5 = 17$.
This is 2 groups of 8 and 1 left over. Write 1, carry 2 and write it in the next column.

$275_8 \times 6 = 2156_8$.

b)

$$\begin{array}{r} 5\quad 2 \\ 5\overline{)\,3\ \ {}^{2}6{}^{2}\ \ {}^{1}0{}^{2}} \end{array}$$

5 will not divide into 3. Carry 3 groups of 8.
$3 \times 8 = 24$ $\quad$ $24 + 2 = 26$.
Now divide 5 into 26. The answer is 5 and there is one left over. Write 5, carry the one group of 8.
$8 + 2 = 10$. $\quad$ Divide 5 into 10. The answer is 2, write 2.

$322_8 \div 5 = 52_8$.

Try this exercise.

Exercise F

Calculate the following in base 8.

1. $234_8 \times 4$
2. $1135_8 \times 3$
3. $2245_8 \times 5$
4. $1035678_8 \times 3$
5. $23461_8 \times 2$
6. $232_8 \div 7$
7. $2550_8 \div 4$
8. $5423_8 \div 5$
9. $33360_8 \div 4$
10. $22726_8 \div 2$

Check your answers with those at the end of the unit.

The hexadecimal (hex) system

This is a system to base 16. The sixteen digits used are,

Ø 1 2 3 4 5 6 7 8 9 A B C D E F

where A = 10, B = 11, C = 12 and so on to F = 15. We can add, subtract, mulitiply and divide as for bases 2 and 8, although this requires knowledge of the 16 times table!

It is often necessary to change from the hexadecimal to the binary system.

Study this example.

Example 9

Change $359B_{16}$ into a binary number.

We split up the hex number, treat each digit to base 10 and then give each digit it's binary number, using four digits.

$3_{10} = ØØ11_2$, $5_{10} = Ø1Ø1_2$, $9_{10} = 1ØØ1_2$, $B = 11_{10} = 1Ø11_2$,

hex	3 / 5 / 9 / B
binary	ØØ11 / Ø1Ø1 / 1ØØ1 / 1Ø11

We can now write $359B_{16}$ in binary form. We may ignore the first two zeros.

$359B_{16} = 11Ø1Ø11ØØ11Ø11_2$.

Try this exercise.

Exercise G

Convert the following hex numbers into binary numbers.

1. 28A
2. 135D
3. A14
4. BØ88
5. C23A
6. 113B
7. 763
8. 9BBA
9. 352Ø
10. 11FD

Check your answers with those at the end of the unit.

Conversion from the octal system to the binary system

Consider this example.

Example 10
Change 4357_8 into a binary number.

We split up the octal number, treat each digit to base 10 and then give each digit its binary number, using **three** digits this time.
$4_{10} = 100_2$, $3_{10} = 011_2$, $5_{10} = 101_2$, $7_{10} = 111_2$,

octal	4 / 3 / 5 / 7
binary	110 / 011 / 101 / 111

We can now write 4357_8 in binary form.

$$4357_8 = 100011101111_2.$$

Try this exercise.

Exercise H
Convert the following octal numbers into binary numbers.

1. 27_8	6. 113_8
2. 135_8	7. 763_8
3. 134_8	8. 4210_8
4. 3024_8	9. 3520_8
5. 263_8	10. 115_8

Check your answers with those at the end of the unit.

Negative numbers

Twos complement: quick method

Study this example.

Example 11
Find the 8-bit number for -84_{10} in binary form.

First write the 8-bit (8 digit) number for 84_{10} in binary form, Ø1Ø1Ø1ØØ_2.
Starting at the right hand side, rewrite the number up to and including the first 1, **Ø1Ø1Ø1ØØ**, write, 1ØØ.
For all the remaining digits, **Ø1Ø1Ø**1ØØ, the Ø's become 1 and the 1's become Ø, as follows, **1Ø1Ø1**1ØØ.
$-84_{10} = \text{1Ø1Ø11ØØ}_2$.

Ones complement
In the binary system, the ones complement method for finding a negative number is to subtract each digit from 1.
Study this example.

Example 12
Find the ones complement of 84_{10}, Ø1Ø1Ø1ØØ_2.

$11111111_2 - \text{Ø1Ø1Ø1ØØ}_2 = \text{1Ø1Ø1Ø11}_2$.

Twos complement: formal method
To find the twos complement we add 1 to the ones complement.

Study the example.

Example 13
Find the twos complement of 84_{10}, Ø1Ø1Ø1ØØ_2.

We add 1 to the ones complement.
$\text{1Ø1Ø1Ø11}_2 + 1_2 = \text{1Ø1Ø11ØØ}_2$.
As in Example 11, $-84_{10} = \text{1Ø1Ø11ØØ}_2$.

The explicit sign/sign and magnitude method

Study this example.

Example 14

Express

a) 15,

b) -15,

in binary form using the explicit sign method and 8 bits.

a) In binary form, 15 = 1111.

We now use 7 digits for the actual number,

15 = ØØØ1111.

We write a Ø in front of this 7 digit number to show it is positive, so that,

$15 = ØØØØ1111_2$ as an 8 bit binary number.

b) We follow the same process as above until we have the seven digit number for 15,

15 = ØØØ1111,

then we write 1 in front of this number to show that it is negative,

$-15 = 1ØØØ1111_2$.

Try this exercise.

Exercise I

For each of the following denary numbers, write in 8-bit binary form,

a) the ones complement,

b) the twos complement,

1. 28
2. 115
3. 64
4. 78
5. 20
6. 56

Write each of the following numbers as an 8 bit binary number using the explicit sign method.

7. 23
8. -23
9. 13
10. -13
11. 35
12. -35
13. 4
14. -4

Check your answers with those at the end of the unit.

Note that some calculators can be used in base N mode and will do all of these calculations for you. An example of a calculator capable of this is the Casio *fx* 6300G. Check your calculator instruction booklet to see if your calculator has this facility.

Answers

Exercise A

1. 20
2. 68
3. 52
4. 60
5. 116
6. 113
7. 13
8. 114
9. 84
10. 217

Exercise B

1. 1101
2. 1100101
3. 10011011
4. 11011
5. 1100100
6. 10111
7. 11110
8. 100000
9. 101101
10. 11010100

Exercise C

1. 11010100
2. 10011100
3. 10100100
4. 10001000
5. 10000001
6. 100101
7. 110100
8. 101101
9. 1011101
10. 11010

Exercise D

1. 1010111
2. 1011111
3. 1011100
4. 111100
5. 100100110
6. 1110
7. 1001
8. 1101
9. 10100
10. 11111

Exercise E

1. 7077_8
2. 6211_8
3. 7214_8
4. 5677_8
5. 36063_8
6. 52117_8
7. 3555_8
8. 277_8
9. 22423_8
10. 43706_8

Exercise F

1. 1160_8
2. 3427_8
3. 13471_8
4. 313145_8
5. 47142_8
6. 26_8
7. 532_8
8. 1067_8
9. 6674_8
10. 11353_8

Exercise G

1. 1010001010_2
2. 10011010111101_2
3. 1010000010100_2
4. 1011100001000010000_2
5. 11000010000111010_2
6. 1000100111011_2
7. 11101100011_2
8. 10011011101111010_2
9. $1101010010000 0_2$
10. 1000111111101_2

Exercise H

1. 10111_2
2. 1011101_2
3. 1011100_2
4. 11000010100_2
5. 10110011_2
6. 1001011_2
7. 111110011_2
8. 1000100001000_2
9. 11101010000_2
10. 1001101_2

Exercise I

1. a) 11100011_2
   b) 11100100_2
2. a) 10001100_2
   b) 10001101_2
3. a) 10111111_2
   b) 11000000_2
4. a) 10110001_2
   b) 10110010_2
5. a) 11101011_2
   b) 11101100_2
6. a) 11000111_2
   b) 11001000_2
7. 00010111_2
8. 10010111_2
9. 00001101_2
10. 10001101_2
11. 00100011_2
12. 10100011_2
13. 00000100_2
14. 10000100_2