Unit 43

Set theory

Objectives

On completion of this unit you should understand:

1. Set notation.

2. Equality and equivalence

3. Union and intersection.

4. Venn diagrams.

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Sets

A set is a collection of objects, numbers or ideas. The contents of the set are called the elements of the set.

Consider these examples.

Example 1

Set A contains all the even numbers between zero and 11. Write the members of the set.

We write these in curly brackets as shown below.

A = {2, 4, 6, 8, 10}.

Example 2

Write an expression for the set of positive, odd numbers, B.

B = {1, 3, 5, 7,............} or B = { all the positive odd numbers}

Example 3

Write an expression for the set C, for which C = $\{x: 3 \leq x \leq 9\}$

This expression tells us that the elements of the set are the numbers 3 to 9 inclusive.

C = { 3, 4, 5, 6, 7, 8, 9}

A **finite** set is one in which all the elements are listed such as set C in Example 3.

Set B in Example 2 is an **infinite** set. It is impossible to list all the elements.

The **null** or empty set is shown as Ø or {}. An example of a null set would be,

D = {all the triangles with six sides},

then,

D = Ø

or,

D = {}.

Try the exercise on the next page.

Exercise A

Write an expression for each of the following.

1. Set A contains the first 8 letters of the alphabet.
2. Set B, for which B = $\{x: 2 < x < 11\}$.
3. Set C, the set of positive, even numbers.
4. Set D, all the prime numbers between 6 and 14, if a prime number is one which will only divide by itself or 1, leaving no remainder.
5. Set E, which contains all the positive multiples of 2 up to and including 12.

Check your answers with those at the end of the unit.

The universal set

The universal set, which is usually denoted by the letter $\mathcal{E}$, is the set, which contains all the available elements for the problem.

For example, we may have a universal set $\mathcal{E}$ as follows,

$$\mathcal{E} = \{4, 5, 6, 7, 8, 9, 11\}$$

then we may have subsets of $\mathcal{E}$.

If A = $\{4, 6, 8\}$ and B = $\{5, 7, 8, 11\}$ they both contain some elements of $\mathcal{E}$, so they are subsets of $\mathcal{E}$.

We write that $A \subset \mathcal{E}$ and $B \subset \mathcal{E}$.

$\subset$ means 'is a subset of'. **$\not\subset$ means 'is not a subset of'.**

Study this example.

Example 4

If $\mathcal{E} = \{5, 7, 10, 15, 17, 19\}$ state whether the following are subsets of $\mathcal{E}$,

a) A = $\{5, 15, 19\}$,
b) B = $\{10, 17, 24\}$,
c) C = $\{8\}$.

a) All the elements of A are in $\mathcal{E}$, so A is a subset of $\mathcal{E}$. $A \subset \mathcal{E}$.

b) 24 is not a member of $\mathcal{E}$, so B is not a subset of $\mathcal{E}$. $B \not\subset \mathcal{E}$.

c) 8 is not a member of $\mathcal{E}$, so C is not a subset of $\mathcal{E}$. $C \not\subset \mathcal{E}$.

Try this exercise.

Exercise B

For each of the following questions, the universal set is,

$\mathcal{E} = \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

1. Write A, the subset of $\mathcal{E}$, which contains even numbers less than 10.
2. Write B, the subset of $\mathcal{E}$, which contains the numbers, which are multiples of 4.
3. Write C, the subset of $\mathcal{E}$, which contains prime numbers only.
4. Write D, the subset of $\mathcal{E}$, which contains multiples of 5.
5. Write F, the subset of $\mathcal{E}$, which contains factors of 40.

Check your answers with those at the end of the unit.

Study this example.

Example 5

State whether each of the following statements is true or false.

a) $7 \in$ {multiples of 2},
b) $5 \notin$ {factors of 10},
c) triangle $\in$ {polygons}.

$\in$ means 'is a member of', **$\notin$ means 'is not a member of'.**

a) False, 7 is not a multiple of 2.

b) False, 5 is a factor of 10.

c) True, a triangle is a three sided polygon.

Try this exercise.

Exercise C

State whether each of the following statements is true or false.

1. $3 \in$ {multiples of 6}
2. $7 \notin$ {factors of 14}
3. square $\in$ {four sided polygons}
4. $a \in$ {letters of the alphabet}
5. $9 \notin$ {prime numbers}
6. $10 \in$ {even numbers}
7. $14 \in$ {even numbers less than 10}
8. $5 \notin \{x: 0 < x < 5\}$
9. $12 \in$ {numbers divisible by 3}
10. $15 \in$ {multiples of 2 up to and including 16}

Check your answers with those at the end of the unit.

The complement of a set

Consider this example.

Example 6

If $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 2, 5, 6, 8\}$, write the complement of A. **The complement of A is written as A'.**

A' contains all the elements of $\mathcal{E}$, which are not in A, so,

$A' = \{3, 4, 7\}$.

Equality and equivalence

The order of the elements in the set does not matter although it is usual to write numbers in ascending order. We can say however that if,

$A = \{a, b, c, d,\}$ and $B = \{d, b, a, c\}$ then the sets are equal and

$$A = B.$$

If two sets have the same number of elements, they are said to be equivalent. If, $A = \{a, b, c, d,\}$ and $C = \{q, p, h, g,\}$ then, the sets are equivalent, the number of elements in A is the same as the number of elements in C, we write,

$$n(A) = n(C) = 4.$$

Try this exercise.

Exercise D

$\mathcal{E}$ = {all the letters of the alphabet from *a* to *h* inclusive.}

$A = \{a, b, c, d, e\}$ $B = \{a, b, c\}$ $C = \{d, e, f\}$ $D = \{c, a, b\}$

Copy and complete the following statements.

1. $n(B) = n(....) = n(....)$.
2. $B \subset .....$
3. $D = ...$
4. $A' = ...$
5. $n(A') = ...$

Check your answers with those at the end of the unit.

Union and intersection

Consider these examples.

Example 7

If A = {2, 3, 4, 5, 10} and B = {3, 4, 11, 12} write,

a) $A \cup B$,
b) $A \cap B$.

a) $\cup$ means the union of the sets. We write all the elements of both sets,
$A \cup B = \{2, 3, 4, 5, 10, 11, 12\}$.

b) $\cap$ means the intersection of the sets. We write the members, which are common to both sets,
$A \cap B = \{3, 4\}$.

Example 8

If, $\mathcal{E} = \{x: 0 < x < 16\}$, $A = \{3, 5, 7, 9, 10\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{5, 9, 10, 15\}$, find each of the following,

a) $A \cup B \cup C$,
b) $A \cap B \cap C$,
c) $(A \cup B \cup C)'$,
d) $(A \cup B)' \cap C$.

a) $A \cup B \cup C$ is the set containing all the elements of A, B and C.
$A \cup B \cup C = \{3, 5, 6, 7, 9, 10, 12, 15\}$.

b) $A \cap B \cap C$ is the set containing the elements which are common to all three sets.
$A \cap B \cap C = \{9\}$.

c) $(A \cup B \cup C)'$, we write the complement of $A \cup B \cup C$.
$(A \cup B \cup C)' = \{1, 2, 4, 8, 11, 13, 14\}$.

d) $(A \cup B)' \cap C$,
$A \cup B = \{3, 5, 6, 7, 9, 10, 12, 15\}$
so,
$(A \cup B)' = \{1, 2, 4, 8, 11, 13, 14\}$
and
$C = \{5, 9, 10, 15\}$
so there are no elements common to both $(A \cup B)'$ and C,
$(A \cup B)' \cap C = \{\}$.

Try this exercise.

Exercise E

$\mathcal{E} = \{x: 5 < x \leq 20\}$, $A = \{6, 9, 12, 15, 18\}$, $B = \{12, 14, 16, 18\}$ and $C = \{7, 8, 9, 14, 19\}$, find each of the following.

1. $A \cup B$
2. $B \cap C$
3. $(A \cup B)'$
4. $A \cup B \cup C$
5. $A \cap B \cap C$
6. $(A \cup B)' \cap C$
7. $A \cap B$
8. $(A \cap B \cap C)'$
9. $n(A \cap B \cap C)'$
10. $(B \cap C) \cup A$

Check your answers with those at the end of the unit.

Venn diagrams

In a Venn diagram, the universal set is represented by a rectangle, and subsets of this are then represented by closed curves.
Consider this example.

Example 9

If $\mathcal{E} = \{x: 0 \le x \le 10\}$, $A = \{1, 2, 4, 7\}$, $B = \{1, 5, 7, 9,\}$ and $C = \{3, 4, 5, 7, 10\}$, represent this on a Venn diagram.

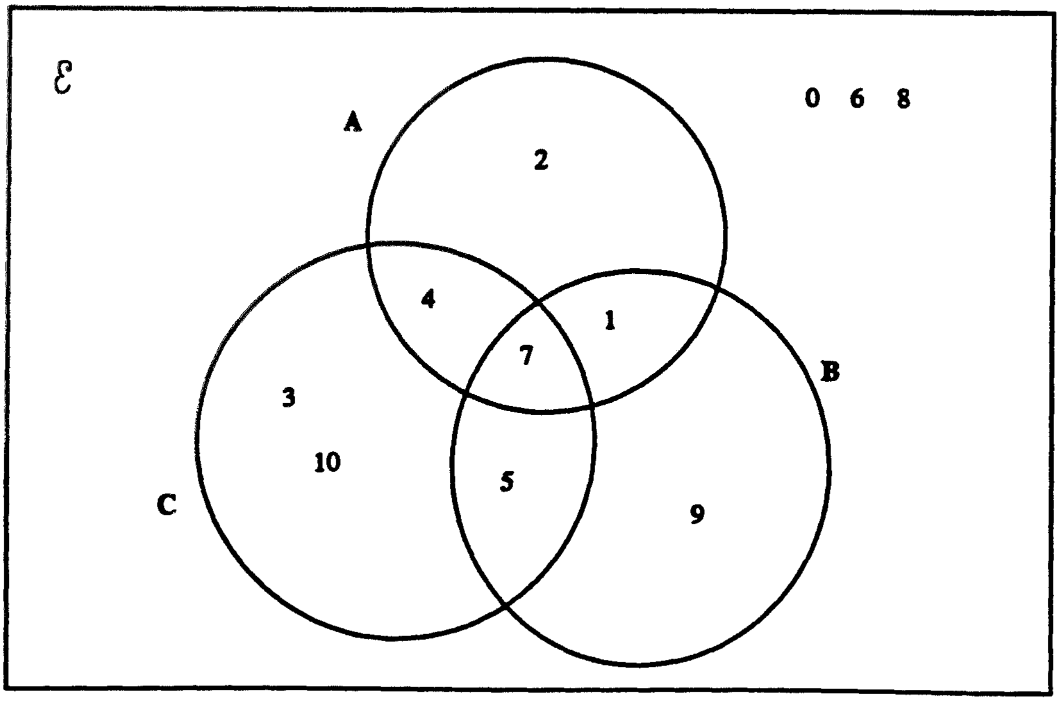
The elements of the universal set are the numbers 0 to 10 inclusive. We draw a rectangle to represent the universal set and the three intersecting curves for the sets A, B and C.

The area common to all three curves must contain the element common to all three sets. We put 7 into this area. $A \cap B \cap C = \{7\}$.

1 is common to both A and B and therefore goes into the intersection for A and B. Similarly, 5 is common to both B and C and we put this into the intersection of B and C.

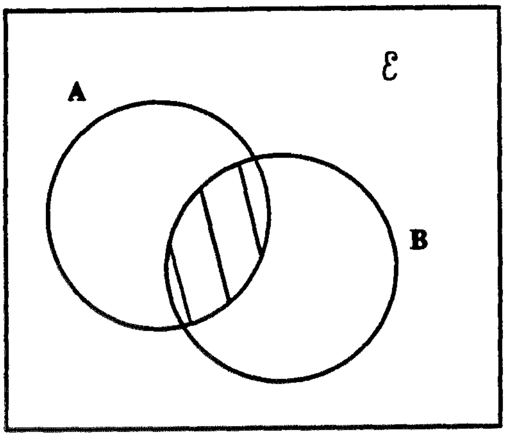
We proceed in this manner, for example, 2 is only in set A and is therefore in the section of the curve for A only.

Once the elements of the three sets have been placed, we consider which numbers from the universal set have not been used. These are 0, 6 and 8. These are placed outside the curves but inside the rectangle.

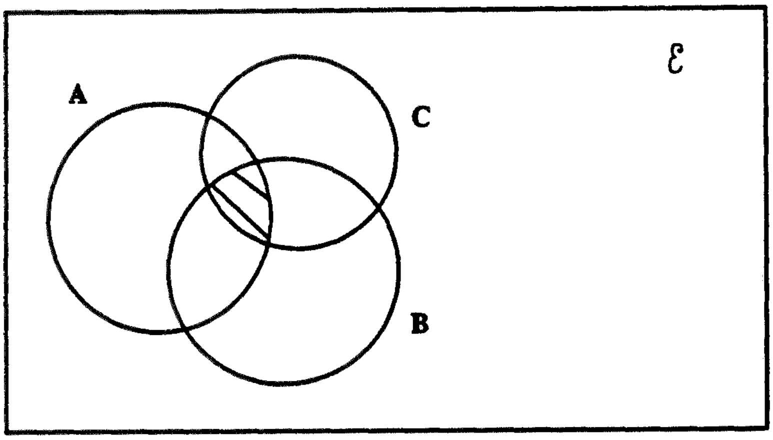


Example 10
Use set notation to describe the shaded area on the following diagrams.

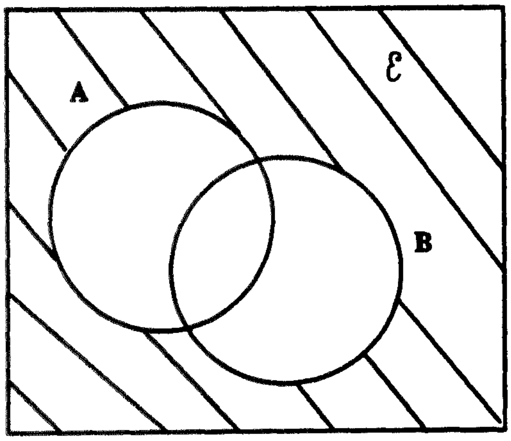
a)



b)



c)



a) $A \cap B$.

b) $A \cap B \cap C$.

c) $(A \cup B)'$.

Try this exercise.

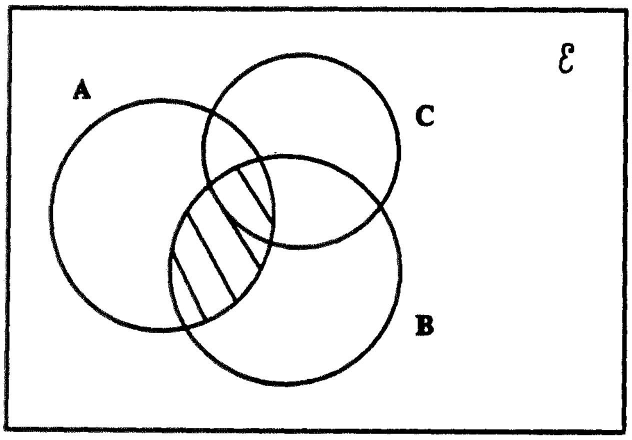
Exercise F

1. If $\mathcal{E} = \{x: 3 \le x \le 14\}$, P = {5, 9, 10, 14, }, Q = {9, 10 13,14,} and R = {3, 5, 10, 14,}, represent this on a Venn diagram.

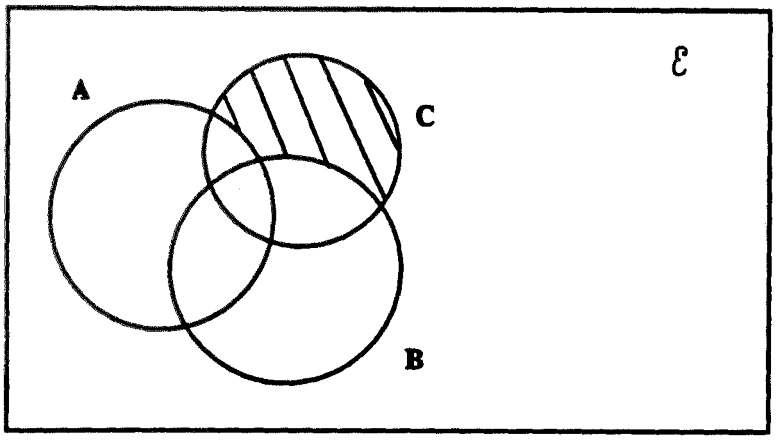
2. If $\mathcal{E} = \{x: 20 \le x \le 40\}$, A = {20, 25, 30, 35, 40}, B = {20, 30, 40}, and C = {20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40}, represent this on a Venn diagram.

Use set notation to describe the shaded area on the Venn diagrams below.

3.



4.



Check your answers with those at the end of the unit.

Answers

Exercise A

1. A = {a, b, c, d, e, f, g, h}
2. B = {3, 4, 5, 6, 7, 8, 9, 10}
3. C = {2, 4, 6, 8,.....}
4. D = {7, 11, 13}
5. E = {2, 4, 6, 8, 10, 12}

Exercise B

1. A = {4, 6, 8}
2. B = {4, 8, 12, 16, 20}
3. C = { }
4. D = {10, 20}
5. F = {4, 8, 10, 20}

Exercise C

1. False
2. False
3. True
4. True
5. True
6. True
7. False
8. True
9. True
10. False

Exercise D

1. n(B) = n(C) = n(D) = 3
2. B $\subset$ A or B $\subset \mathcal{E}$
3. D = B
4. A' = {f, g, h}
5. 3

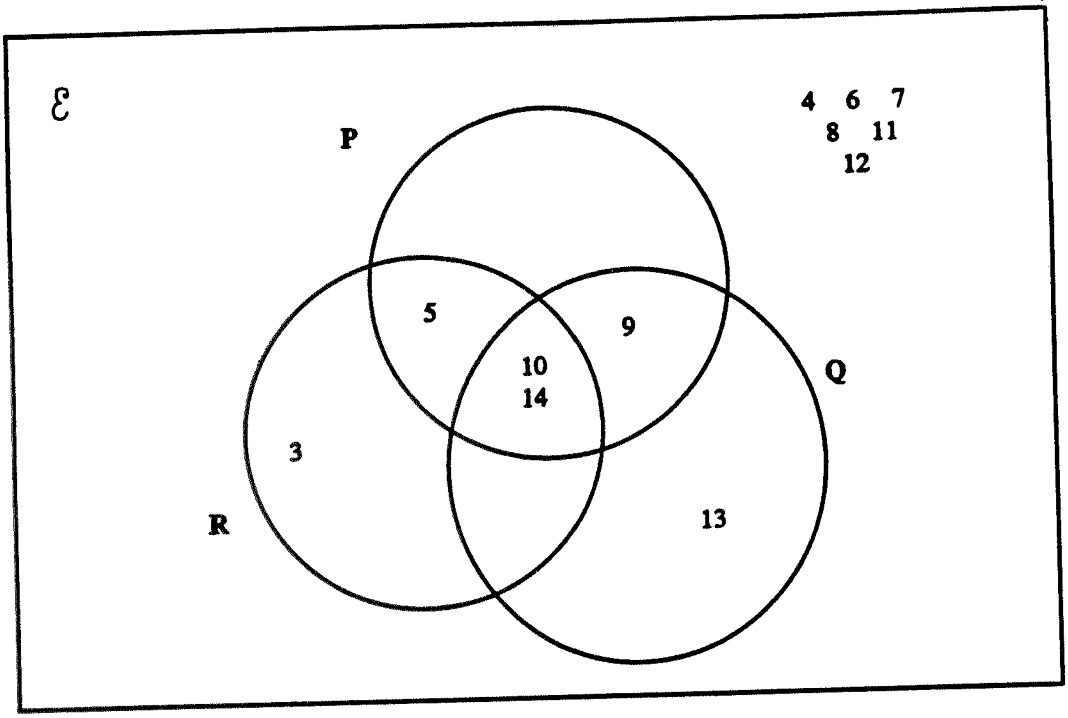
Exercise E

1. {6, 9, 12, 14, 15, 16, 18}
2. {14}
3. {7, 8, 10, 11, 13, 17, 19, 20}
4. {6, 7, 8, 9, 12, 14, 15, 16, 18, 19}
5. {}
6. {7, 8, 19}
7. {12, 18}
8. {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20} = $\mathcal{E}$
9. 15
10. {6, 9, 12, 14, 15, 18}

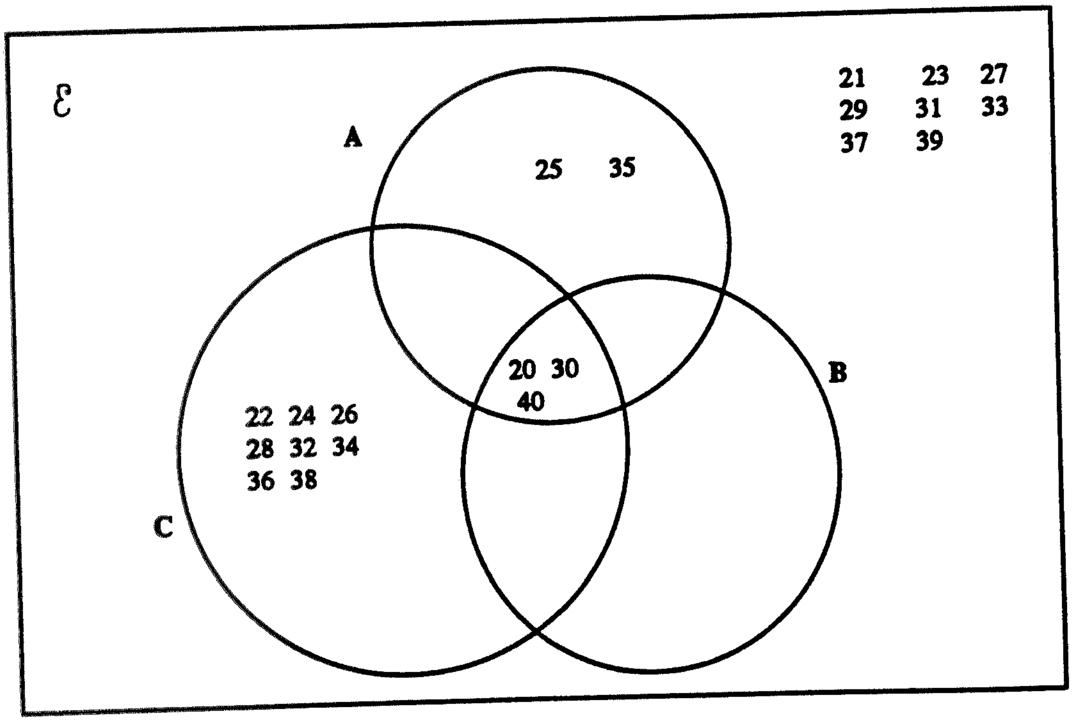
Answers (*Continued*)

Exercise F

1.



2.



3. $A \cap B$
4. $(A \cup B)' \cap C$