MATHS Xtra
ALGEBRA

Unit 3

Transposition

Objectives:

On completion of this unit you should be able to transpose formulas including those with:

1. Brackets.
2. Powers.
3. Roots.

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Transposition of formulas

It is possible that you may be given the formula for say the area of a rectangle and asked to find its length. One way to do this is to transpose the formula to make the length, L, the subject of the formula.

You adjust the formula until the new subject is on its own on one side or other of the formula.

You can add or subtract the same quantity to/from both sides of a formula. You can also multiply or divide both sides by the same quantity.

In general the order to follow is,

get rid of fractions,

try to 'undo' whatever has been done to the new subject,

get rid of brackets as soon as convenient.

Consider the following examples.

Example 1

Given that, for a rectangle, Area = length x breadth,

a) transpose, $A = Lb$, to make L the subject of the formula.

b) Find the length of the rectangle if the breadth is 6cm. and the area is $48cm^2$.

a) $A = Lb$

A is the subject of the formula.

If we wish to transpose (re-arrange) the formula, to make L the subject of the formula, we proceed as follows.

$$A = Lb$$

L is multiplied by b so divide both sides by b.

$$\frac{A}{b} = \frac{Lb}{b}$$

$$L = \frac{A}{b}.$$

b) We can now find the length of the rectangle.

$$L = \frac{48}{6}$$

$$L = 8$$

The length of the rectangle is 8cm.

Example 2

Given that, $x = \frac{y}{z}$, make y the subject of the formula.

y is divided by z, so multiply both sides by z.

$$xz = \frac{yz}{z}$$
$$y = xz.$$

Example 3

The equation of a straight line is,

$$y = mx + c$$

a) Make c the subject of the formula.

b) Find the value of c, if $m = 4$, at the point where $x = 2$ and $y = 5$.

a) c has mx added to it, so subtract mx from both sides.

$$y - mx = mx + c - mx$$
$$y - mx = c$$
$$c = y - mx.$$

b)
$$c = 5 - (4 \times 2)$$
$$c = 5 - 8$$
$$c = -3.$$

Example 4

Using the same equation, $y = mx + c$,

a) make m the subject,

b) find m at the point where $x = 2$ and $y = 4$, if c is equal to 5.

a) mx has c added to it, so subtract c from both sides

$$y - c = mx + c - c$$
$$y - c = mx$$

Now m is multiplied by x, so divide both sides by x

$$\frac{y - c}{x} = \frac{mx}{x}$$
$$m = \frac{y - c}{x}.$$

b)
$$m = \frac{4 - 5}{2}$$
$$m = -0.5.$$

Try this exercise.

Exercise A

1. Given the equation $v = u + at$,
   a) rearrange the equation to make a the subject,
   b) find the value of a if $u = 20$, $v = 35$ and $t = 5$.

2. Given the formula $v = wr$,
   a) make r the subject of the formula,
   b) find the value of r if $v = 16$ and $w = 5$.

3. Using the formula $1.8C = F - 32$,
   a) transpose the equation to make F the subject,
   b) find the value of F if $C = 40$.

4. a) Rearrange the formula $\frac{PV}{T} = C$ to make V the subject.
   b) Find the value of V if $C = 24$, $T = 36$ and $P = 144$.

5. Using the equation $E = IR + v$,
   a) rearrange the formula to make R the subject,
   b) find the value of R, if $E = 9.2$, $I = 2.0$ and $v = 3.6$.

Check your answers with those at the end of the unit.

Study the examples on the next page, which involve brackets.

Example 5

Given that, $x = b(c - a)$, make c the subject of the formula.

First multiply out the brackets.

$$x = bc - ba$$

We require c, so we start by adding ba to both sides.

$$x + ba = bc - ba + ba$$
$$x + ba = bc$$

Divide both sides by b.

$$\frac{x + ba}{b} = \frac{bc}{b}$$
$$c = \frac{x + ba}{b}.$$

Example 6

There is an equation which relates degrees Centigrade to degrees Fahrenheit.

$$C = \frac{5}{9}(F - 32)$$

a) Use this formula to convert 68°F to degrees Centigrade.
b) Transpose the formula to make F the subject.
c) Convert 25°C to degrees Fahrenheit.

a) F = 68, so,

$$C = \frac{5}{9}(68 - 32) = \frac{5}{9}(36)$$
$$68°F = 20°C.$$

b)
$$C = \frac{5}{9}(F - 32)$$

Multiply both sides of the equation by 9.

$$9C = 5(F - 32)$$

Divide both sides by 5.

$$\frac{9C}{5} = F - 32$$

Add 32 to both sides of the equation.

$$\frac{9C}{5} + 32 = F$$
$$F = \frac{9C}{5} + 32.$$

c) Substituting C = 25°,

$$F = \frac{9 \times 25}{5} + 32 = 77$$

25°C is equal to 77°F.

Try this exercise.

Exercise B

1. Rearrange the formula $E = I(R + r)$ to make R the subject.

2. Given the formula $Ft = m(v - u)$,
   a) rearrange this equation to make u the subject.
   b) Find a value for u when $F = 12$, $t = 2$, $m = 0.5$ and $v = 56$.

3. The equation $S = \frac{(u + v)t}{2}$ is used when a body moves with constant acceleration.
   a) Transpose the formula to make v the subject.
   b) Find the value of v if $S = 84$, $u = 8$ and $t = 12$.

4. Given the equation $T = \frac{\lambda(x - 5)}{5}$, which relates to the force in a stretched string.
   a) Find T if $\lambda = 1.5$ and $x = 8$.
   b) Rearrange the formula to make x the subject.
   c) Use the formula from b) to find the value of x if $T = 20$ and $\lambda = 4$.

5. The surface area of a cylinder is given by the formula $A = 2\pi r(r + h)$.
   a) Calculate the value for A if $r = 2.5$cm. and $h = 4.2$cm. Give your answer to the nearest whole number.
   b) Transpose the formula to make h the subject.
   c) Using the formula formed in b) determine the value for h correct to 2 decimal places if $A = 32\text{cm}^2$. and $r = 1.4$cm.

Check your answers with those at the end of the unit.

Roots and Powers

The next example involves roots and powers.

Example 7

The volume of a cylinder is given by the formula,

$$V = \pi r^2 h.$$

a) Find the volume, when the radius, r is equal to 10cm. and the height h, is equal to 20cm.

b) Transpose the formula to make r the subject.

c) Find the value of r to three significant figures if the volume is equal to $300cm^3$., and the height of the cylinder is 20cm.

a) $V = \pi r^2 h$

$V = \pi \times 10^2 \times 20$

$V = 6283.1853$

The volume is $6280cm^3$. to three significant figures.

b) $V = \pi r^2 h$

Divide both sides of the equation by π and by h.

$$\frac{V}{\pi h} = r^2$$

$$r^2 = \frac{V}{\pi h}$$

$$r = \sqrt{\frac{V}{\pi h}}.$$

c) $$r = \sqrt{\frac{300}{\pi \times 20}}$$

Calculator: 300 ÷ π ÷ 20 = √ 2.1850969

The radius of the cylinder is 2.19cm to three significant figures.

Try the exercise on the next page.

Exercise C

1. Given that the area of a circle is represented by,
$A = \pi r^2$.
   a) Find A correct to 3 significant figures, when the radius, $r = 7$cm.
   b) Transpose the formula to make r the subject.
   c) Find the radius of the circle to 3 significant figures, if the area is 200cm^2.

2. Kinetic energy, E, which is measured in Joules is related to the mass of the particle, m kg. and its velocity, v m/s. by the equation,
$E = \frac{1}{2}mv^2$.
   a) Find the kinetic energy if $m = 16$kg. and $v = 9$m/s.
   b) Rearrange the formula to make v the subject.
   c) Calculate the value of v to 2 decimal places if the kinetic energy is 55 Joules and the mass m is 9kg.

3. The theorem of Pythagoras is given by,
$a^2 = b^2 + c^2$.
   a) Transpose the formula to make b the subject.
   b) Find the value of b, if $a = 25$cm. and $c = 7$cm.

4. Given the formula, $2aS = v^2 - u^2$,
   a) transpose the formula to make v the subject,
   b) find the value of v, correct to 2 decimal places, if $a = 10$, $S = 12$ and $u = 8$.

5. A formula giving the energy, in Joules, of a particle, mass m kg. is
$E = mgh + \frac{1}{2}mv^2$.
   a) Calculate the total energy if $m = 20$kg., $g = 10\text{m/s}^2$., $h = 12$m. and $v = 5$m/s.
   b) Rearrange the formula to make v the subject.
   c) Find the value for v if E = 85 Joules, $m = 2$kg., $g = 10\text{m/s}^2$. and $h = 3$m.

Check your answers with those at the end of the booklet.

Study this next example.

Example 8

The time period, in seconds, for one complete oscillation of a pendulum is given by the formula,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

a) Find T, the time period to two decimal places, if the length of the pendulum, $l = 0.3$m. and g, the acceleration due to gravity, is $10ms^{-2}$.

b) Make l the subject of the formula.

c) If the length of the pendulum is adjusted, the time period becomes 2 seconds, find the new length of the pendulum. ($g = 10ms^{-2}$).

a) $$T = 2 \times \pi \times \sqrt{\frac{0.3}{10}}$$

Calculator: 2 x π x (0.3 ÷ 10) √ = 1.0882796

T is equal to 1.09 seconds to two decimal places.

b) $$T = 2\pi\sqrt{\frac{l}{g}}$$

To remove the square root sign, we need to square both sides of the equation.

$$T^2 = 2^2\pi^2 \times \frac{l}{g}$$

Get rid of the denominator.

Multiply both sides of the equation by g.

$$T^2 \times g = 4 \times \pi^2 \times l$$

Divide both sides by $4 \times \pi^2$.

$$\frac{T^2 \times g}{4 \times \pi^2} = \frac{4 \times \pi^2 \times l}{4 \times \pi^2}$$

$$l = \frac{T^2 \times g}{4 \times \pi^2} \quad \text{or} \quad \frac{T^2 g}{4\pi^2}$$

c) $$l = \frac{2^2 \times 10}{4 \times \pi^2}$$

Calculator: 2 x^2 x 10 ÷ 4 ÷ π x^2 = 1.0132118

The length of the pendulum is 1.01m. to two decimal places.

Try this exercise.

Exercise D

1. If one side of a right angled triangle is given by the formula,

$$a = \sqrt{c^2 - b^2}\ .$$

a) Find the value of a if $c = 13$ and $b = 5$.
b) Transpose the formula to make b the subject.
c) Find the value of b if $c = 41$ and $a = 9$.

2. The velocity of a particle is given by the formula,

$$v = \sqrt{\frac{2E}{m}}\ .$$

a) Calculate the value of v if $E = 24$ and $m = 3$.
b) Rearrange the formula to make m the subject.
c) Find the value of m if $E = 32$ and $v = 4$.

3. Given the formula,

$$T = 2\pi\sqrt{\frac{l}{g}}\ .$$

a) Transpose this formula to make g the subject.
b) Calculate the value of g to two decimal places if $T = 0.80$ and $l = 0.159$.

4. The radius of a cylinder is given by the formula,

$$r = \sqrt{\frac{V}{\pi h}}\ .$$

a) Rearrange the formula to make h the subject.
b) Find the value of h, to the nearest whole number if $V = 136$ and $r = 3$.

Check your answers with those given at the end of this unit.

The next examples show the procedure if the new subject occurs twice in the equation.

Example 9

Transpose the following equation to make v the subject.

$$4(u + v) = 3(5u - 2v)$$

Multiply out the brackets.	$4u + 4v = 15u - 6v$
Add $6v$ to both sides.	$4u + 4v + 6v = 15u$
Subtract $4u$ from both sides.	$4v + 6v = 15u - 4u$
	$10v = 11u$
Divide both sides by 10.	$v = \frac{11u}{10}$.

Example 10

Tranpose the following equation to make x the subject.

$$y = \frac{x - 3}{2 + x}$$

Multiply both sides of the equation by $(2 + x)$

$$y(2 + x) = \frac{x - 3}{2 + x} \times (2 + x)$$

Cancel the $(2 + x)$ terms on the right hand side.

$$y(2 + x) = x - 3$$

Multiply out the brackets.

$$2y + xy = x - 3$$

Subtract x from both sides.

$$2y + xy - x = x - 3 - x$$

$$2y + xy - x = -3$$

Subtract $2y$ from both sides.

$$2y + xy - x - 2y = -3 - 2y$$

$$xy - x = -3 - 2y$$

Take x out as a common term on the left hand side.

$$x(y - 1) = -3 - 2y$$

Divide both sides by $(y - 1)$.

$$\frac{x(y - 1)}{(y - 1)} = \frac{-3 - 2y}{(y - 1)}$$

Cancel the terms in $(y - 1)$ on the left hand side of the equation.

$$x = \frac{-3 - 2y}{y - 1}.$$

Try this last exercise.

Exercise E

1. Transpose the following equation to make x the subject.

$$3(2x - y) = 5(3y - x)$$

2. Transpose the following to make t the subject.

$$s = \frac{7 - t}{2t + 4}$$

3. Make m the subject of the following equation.

$$2p(m - 3) = q(5 - 2m)$$

Check your answers with those given on the next page.

Answers

Exercise A

1. a) $a = \frac{v - u}{t}$ b) 3
2. a) $r = \frac{v}{w}$ b) 3.2
3. a) $F = 1.8C + 32$ b) 104
4. a) $V = \frac{CT}{P}$ b) 6
5. a) $R = \frac{E - v}{I}$ b) 2.8

Exercise B

1. $R = \frac{E - Ir}{I}$
2. a) $u = \frac{mv - Ft}{m}$ b) 8
3. a) $v = \frac{2S - ut}{t}$ b) 6
4. a) 0.9
   b) $x = \frac{5T + 5\lambda}{\lambda}$ or $\frac{5(T + \lambda)}{\lambda}$
   c) 30
5. a) $105cm^2$. b) $h = \frac{A - 2\pi r^2}{2\pi r}$
   c) 2.24cm.

Exercise C

1. a) $154cm^2$. b) $r = \sqrt{\frac{A}{\pi}}$
   c) 7.98cm.
2. a) 648 Joules b) $v = \sqrt{\frac{2E}{m}}$
   c) 3.50m/s.
3. a) $b = \sqrt{a^2 - c^2}$ b) 24cm.
4. a) $v = \sqrt{2aS + u^2}$ b) 17.44
5. a) 2650 Joules
   b) $v = \sqrt{\frac{2E - 2mgh}{m}}$
   c) 5m/s.

Exercise D

1. a) 12 b) $b = \sqrt{c^2 - a^2}$
   c) 40
2. a) 4 b) $m = \frac{2E}{v^2}$
   c) 4
3. a) $g = \frac{4\pi^2 l}{T^2}$ b) 9.81
4. a) $h = \frac{V}{\pi r^2}$ b) 5

Exercise E

1. $x = \frac{18y}{11}$
2. $t = \frac{7 - 4s}{2s + 1}$
3. $m = \frac{6p + 5q}{2p + 2q}$