



Unit 2

Powers and Multiplication

Objectives:

On completion of this unit you should understand:

1. Use of the calculator power button.
2. How powers are combined.
3. Multiplication of brackets.

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Powers

A power or index is a quick way of writing the multiplication of a number by itself several times. So that

3^4 is a way of writing $3 \times 3 \times 3 \times 3$.

We say '3 to the power 4'.

$$3^4 = 81$$

You should be able to find a power button on your calculator.

It looks like this \times^y

Consider this example.

Example 1

Use your calculator to find the value of,

- a) 2^{10} ,
- b) 3^2 ,
- c) 9^5 ,
- d) 8^4 .

a) **Calculator:** $2 \times^y 10 = 1024$
 $2^{10} = 1024.$

b) **Calculator:** $3 \times^y 2 = 9$
 $3^2 = 9.$

c) **Calculator:** $9 \times^y 5 = 59049$
 $9^5 = 59049.$

d) **Calculator:** $8 \times^y 4 = 4096$
 $8^4 = 4096.$

Try this short exercise using your calculator.

Exercise A

Find the value of each of the following.

- | | |
|-----------|-----------|
| 1. 6^4 | 6. 3^8 |
| 2. 12^3 | 7. 9^4 |
| 3. 2^7 | 8. 5^4 |
| 4. 4^5 | 9. 4^4 |
| 5. 2^9 | 10. 2^6 |

Check your answers with those at the end of the unit.

Indices of algebraic terms

We shall now continue to consider algebraic terms.

a^2 means $a \times a$.

We say 'a to the power 2' or 'a squared'.

a^3 means $a \times a \times a$.

We say 'a to the power 3' or 'a cubed'.

$3c^2$ means $3 \times c \times c$,

but,

$(3c)^2$ means $3c \times 3c = 3 \times c \times 3 \times c = 9 \times c \times c$.

The raised number is called the power or index. (The plural of index is indices).

It is often convenient to write long expressions using indices.

Consider the next example.

Example 2

If,

$$a = 2, b = 3 \text{ and } c = 4,$$

find the value of,

a) a^2b ,

b) $\frac{cb^2}{a}$,

c) $a^3 + b^2$,

d) $(5a)^3$.

a) $a^2b = 2 \times 2 \times 3 = 12$.

b) $\frac{cb^2}{a} = \frac{4 \times 3 \times 3}{2} = 18$.

c) $a^3 + b^2 = 2 \times 2 \times 2 + 3 \times 3$
 $= 8 + 9$
 $= 17$.

d) $(5a)^3 = (5 \times 2) \times (5 \times 2) \times (5 \times 2)$
 $= 10 \times 10 \times 10$
 $= 1000$.

Calculator: $10 \times 3 = 1000$

Now try this exercise.

Exercise B

Rewrite, using powers.

1. $m \times m \times m \times n$
2. $6 \times c \times c \times d \times d \times d$
3. $4 \times p \times p \times 2 \times q \times q \times q$
4. $t \times t \times 3 \times s \times s \times s \times s \times 5 \times r \times r \times r$
5. $2 \times a \times b \times a \times a \times 3 \times b \times b \times b$

If $x = 2$, $y = 4$ and $z = 5$, write in full and then evaluate the following.

- | | |
|---------------------|------------------------|
| 6. y^3 | 11. $(2y)^3$ |
| 7. $x^2 + z$ | 12. $(4x)^2 + 3z$ |
| 8. $x^4 + xy^3$ | 13. $(2x)^4 + 3xy^3$ |
| 9. $\frac{3yz}{2x}$ | 14. $3(yz)^2$ |
| 10. $(xyz)^4$ | 15. $\frac{xyz}{2x^2}$ |

Check your answers with those shown at the end of the booklet.

Multiplying powers

The next set of examples involve multiplication of terms with powers.

You should remember that,

$$3^4 \times 3^2 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

Similarly,

$$c^3 \times c^4 = c \times c \times c \times c \times c \times c \times c = c^7$$

When multiplying terms of the same number or letter we can **add** the powers.

Example 3

Simplify,

- a) $b^2 \times b^3$,
- b) $x^3 \times x^4$,
- c) $x^4 \times y^3 \times x^5$.

a) $b^2 \times b^3 = b^{(2+3)} = b^5$.

b) $x^3 \times x^4 = x^{(3+4)} = x^7$.

c) $x^4 \times y^3 \times x^5 = x^{(4+5)} \times y^3$
 $= x^9 y^3$.

Dividing Powers

The next set of examples illustrate how to divide terms to a power.

First consider the numbers,

$$2^7 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = \frac{128}{8} = 16$$

This is 2^4 .

Consider this again.

$$2^7 \div 2^3 = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2}} = 2^4$$

Now look at these algebraic examples.

Example 4

Simplify,

a) $h^4 \div h^3$,

b) $x^4 \div x^2$.

a) $h^4 \div h^3 = \frac{h \times h \times h \times h}{h \times h \times h} = h$

Note that h is the same as h^1 .

b) $x^4 \div x^2 = \frac{x \times x \times x \times x}{x \times x} = x^2$.

You should have noticed from these examples that when dividing terms with the same number or letter we can **subtract** the powers.

Example 5

Simplify,

a) $y^5 \div y^3$,

b) $q^6 \div q^2$.

a) $y^5 \div y^3 = y^{(5-3)} = y^2$.

b) $q^6 \div q^2 = q^{(6-2)} = q^4$.

Try the exercise on the next page.

Exercise C

Simplify the following.

1. $p^5 \times p^3$
2. $y^3 \times y^2 \times y$
3. $t^2 \times t^5 \times s^3 \times s^2$
4. $3 \times m \times n^2 \times 4 \times m^4 \times n^3$
5. $a^6 \div a^4$
6. $b^{12} \div b^5$
7. $6z^3 \div 2z$
8. $p^2 \times p^7 \div p^3$
9. $m^4 \div m^3 \times m^2$
10. $8t^3 \times 3t^2 \div 2t^4$

Check your answers with those at the end of the booklet.

Multiplication of brackets

We have already multiplied a bracket by a term outside the bracket. You will remember this from Basic Algebra, Unit 1.

For example we can expand $2(3x + 2)$ to give,

$$2(3x + 2) = 6x + 4$$

We multiplied each term inside the bracket by the number outside the bracket.

Now consider these examples.

Example 6

Expand $(x + 3)(x + 2)$.

We need to multiply $(x + 2)$ by x and by 3 .

$$\begin{array}{l} (x + 3)(x + 2) \\ \hline x^2 + 2x + 3x + 6 \\ \hline x^2 + 5x + 6. \end{array}$$

$$\begin{array}{l} x \times x = x^2 \\ x \times 2 = 2x \\ 3 \times x = 3x \\ 3 \times 2 = 6 \end{array}$$

Example 7

Expand $(2x + 3)(x - 1)$.

$$\begin{array}{l} (2x + 3)(x - 1) \\ \hline 2x^2 - 2x + 3x - 3 \\ \hline 2x^2 + x - 3. \end{array}$$

$$\begin{array}{l} 2x \times x = 2x^2 \\ 2x \times (-1) = -2x \\ 3 \times x = 3x \\ 3 \times (-1) = -3 \end{array}$$

These are referred to as quadratic expressions and we shall factorise these in a later unit.

Try this exercise.

Exercise D

Expand the following.

1. $(x + 4)(x + 3)$
2. $(x - 2)(x - 5)$
3. $(x - 4)(x + 6)$
4. $(x + 1)(x - 8)$
5. $(2x - 3)(x + 1)$
6. $(x - 3)(4x - 2)$
7. $(-2x + 3)(3x - 2)$
8. $(5x - 1)(5x + 1)$
9. $(x - 3)(x - 3)$
10. $(2x + 3)^2$

Check your answers with those at the end of the unit.

Answers

Exercise A

1. 1296
2. 1728
3. 128
4. 1024
5. 512
6. 6561
7. 6561
8. 625
9. 256
10. 64

Exercise B

1. m^3n
2. $6c^2d^3$
3. $8p^2q^3$
4. $15t^2s^4r^3$
5. $6a^3b^4$
6. 64
7. 9
8. 144
9. 15
10. 2560000
11. 512
12. 79
13. 640
14. 1200
15. 5

Exercise C

1. p^8
2. y^6
3. t^7s^5 or s^5t^7
4. $12m^5n^5$
5. a^2
6. b^7
7. $3z^2$
8. p^6
9. m^3
10. $12t$

Exercise D

1. $x^2 + 7x + 12$
2. $x^2 - 7x + 10$
3. $x^2 + 2x - 24$
4. $x^2 - 7x - 8$
5. $2x^2 - x - 3$
6. $4x^2 - 14x + 6$
7. $-6x^2 + 13x - 6$
8. $25x^2 - 1$
9. $x^2 - 6x + 9$
10. $4x^2 + 12x + 9$