MATHS Xtra

Unit 19

Trig graphs

Objectives

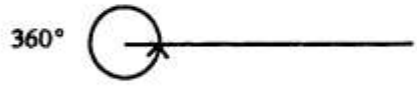
On completion of this unit you should be able to:

1. Recognise and draw graphs of $\sin x$, $\cos x$ and $\tan x$.
2. Use simple translations to transpose the graphs in either the x or y directions.

Cook, M. and Rimmer, H. (1993),
Stafford College, Earl Street, Stafford, ST16 2QR

Graphs of the trig functions

An angle is a measure of rotation.
One complete revolution in an anti-clockwise direction is 360°.



Two complete revolutions in an anticlockwise direction is 720°. (360° x 2)

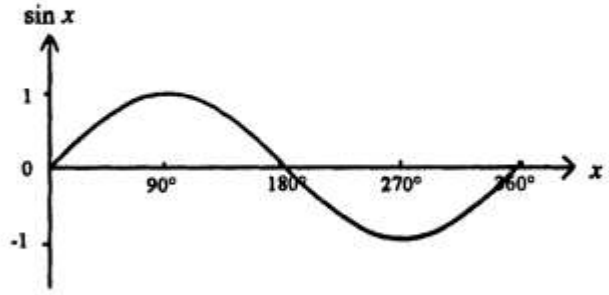
The sine curve

Consider this example.

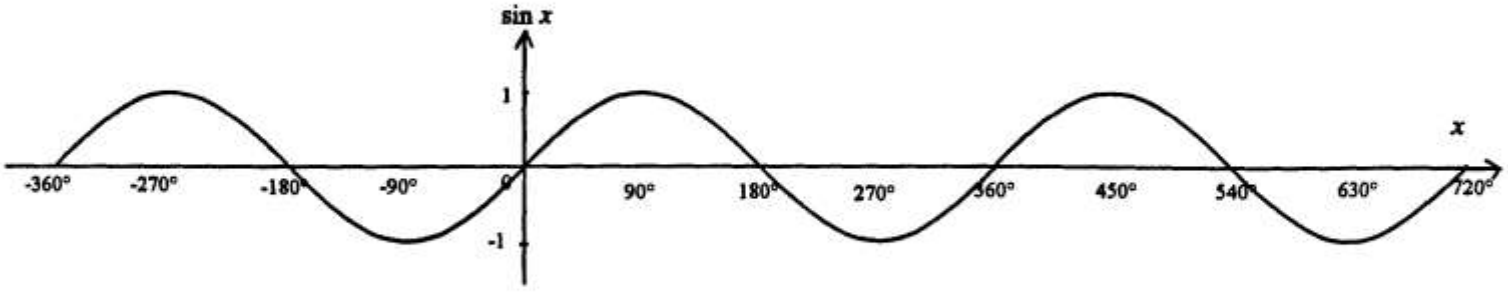
Example 1

Using your calculator, find the values of sin x for values of x from 0° to 360° in 30° intervals. Plot these values on a graph.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
sin x	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0



It can be seen from the graph that for values of x between 0° and 180°, sin x is positive. For values of x between 180° and 360°, sin x is negative. At 0°, 180° and 360°, sin x is zero.
The graph shown in Example 1 is just one cycle of the sine curve. The curve is continuous and repeats itself. Two more cycles have been added in the diagram below.



Try this exercise.

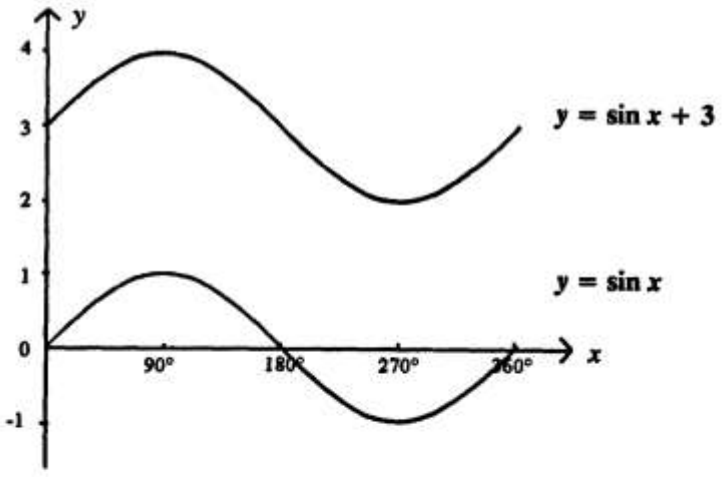
Exercise A

1. Using values for x, from -180° to 540°, find the values of $\sin x$ in 30° intervals. Use these to draw a sine curve on graph paper. Use a scale of 1cm. to represent 30° on the x axis and 2cm. to represent 1 unit on the y axis. You should obtain a curve similar to the one at the bottom of the previous page.
   From the graph, write all the values of x, correct to the nearest degree, for the following values of $\sin x$.
   a) $\sin x = 0.5$
   b) $\sin x = 0.71$
   c) $\sin x = -0.71$
   d) $\sin x = 0.8$
   e) $\sin x = -0.8$
   f) $\sin x = 0.9$
   g) $\sin x = -0.25$
   h) $\sin x = -0.5$

Check your answers with those at the end of the unit.
Now consider this example.

Example 2

On the same axes, using values of x from 0° to 360°, draw the graphs of,
   a) $y = \sin x$,
   b) $y = \sin x + 3$.



To draw the graph of $y = \sin x + 3$, each value of $\sin x$ has 3 added to it. This transposes the graph of $y = \sin x$ by moving it 3 points in the positive y direction.

Try this exercise.

Exercise B

1. Using values of x from 0° to 360°, on the same axes draw the graphs of,

$$y = \sin x + 2,$$
$$y = \sin x - 3.$$

What translation transposes the graph of $y = \sin x + 2$, onto the graph of $y = \sin x - 3$?

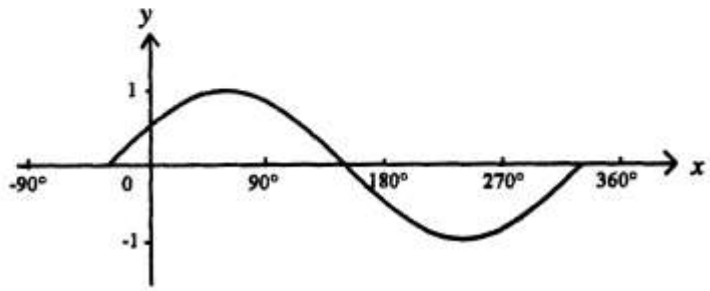
Check your answers with those at the end of the unit.
Study this example.

Example 3

Using values from -30° to 330°, draw the curve for,

$$y = \sin(x + 30°).$$

x	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$x + 30°$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin x$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0



You can see that by plotting $y = \sin(x + 30°)$, the curve for $y = \sin x$ has been transposed in the x direction. It has been transposed by 30° in the negative x direction.

Try the exercise on the next page.

Exercise C

1. Draw the curve for $y = \sin(x - 60°)$ for values of x from 60° to 420°. Name the translation required to transpose the curve $y = \sin x$ to $y = \sin(x - 60°)$.

2. Draw the curve for $y = \sin(x + 60°)$ for values of x from -60° to 300°. Name the translation required to transpose the curve $y = \sin x$ to $y = \sin(x + 60°)$.

3. Draw the curve for $y = \sin(x - 45°)$ for values of x from 45° to 405°. Name the translation required to transpose the curve $y = \sin x$ to $y = \sin(x - 45°)$.

4. Draw the curve for $y = \sin(x + 45°)$ for values of x from -45° to 315°. Name the translation required to transpose the curve $y = \sin x$ to $y = \sin(x + 45°)$.

Check your answers with those at the end of the unit.

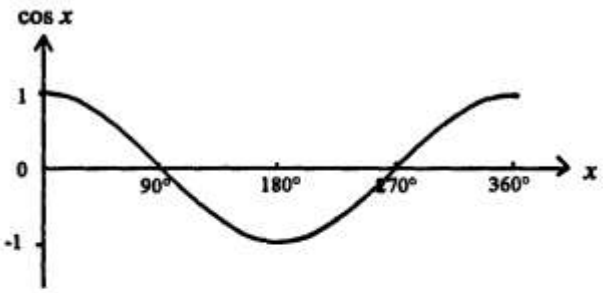
The cosine curve

Study this example.

Example 4

Using your calculator, find values of cos x for values of x from 0° to 360° in 30° intervals. Plot these on a graph.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cos x	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



This is one cycle of the cosine curve. This graph is continuous in the same way as the curve for sin x.

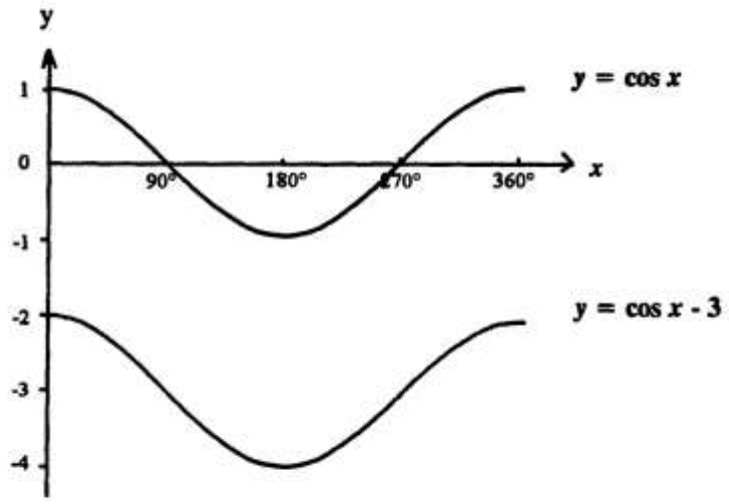
Study these examples.

Example 5

On the same axes, using values of x from 0° to 360°, draw the graphs of,

a) $y = \cos x$,

b) $y = \cos x - 3$.

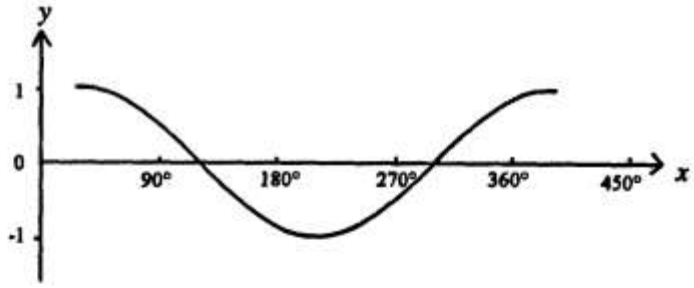


The graph of $y = \cos x - 3$, is a translation of the graph $y = \cos x$ by 3 points in the negative y direction.

Example 6

Using values from 30° to 390°, draw the curve for,

$y = \cos(x - 30°)$.



You should notice that the curve for $y = \cos x$ has been transposed 30° in the positive x direction to obtain the curve for $y = \cos(x - 30°)$.

Try the exercise on the next page.

Exercise D

1. Using values for x, from -180° to 540°, find the values of cos x in 30° intervals. Use these to draw a cosine curve on graph paper. Use a scale of 1cm. to represent 30° on the x axis and 2cm. to represent 1 unit on the y axis. From the graph, write all the values of x, correct to the nearest degree for the following values of cos x.
   a) $\cos x = 0.5$
   b) $\cos x = 0.71$
   c) $\cos x = -0.71$
   d) $\cos x = 0.8$
   e) $\cos x = -0.8$
   f) $\cos x = 0.9$
   g) $\cos x = -0.25$
   h) $\cos x = -0.5$

2. Draw the curve for $y = \cos(x - 60°)$ for values of x from 60° to 420°. Name the translation required to transpose the curve $y = \cos x$ to $y = \cos(x - 60°)$.

3. Draw the curve for $y = \cos(x + 60°)$ for values of x from -60° to 300°. Name the translation required to transpose the curve $y = \cos x$ to $y = \cos(x + 60°)$.

4. Draw the curve for $y = \cos(x - 45°)$ for values of x from 45° to 405°. Name the translation required to transpose the curve $y = \cos x$ to $y = \cos(x - 45°)$.

5. Draw the curve for $y = \cos(x + 45°)$ for values of x from -45° to 315°. Name the translation required to transpose the curve $y = \cos x$ to $y = \cos(x + 45°)$.

6. Draw the curve for $y = -\cos x$ for values of x from 0° to 360°. Name the translation required to transpose the curve $y = \cos x$ to $y = -\cos x$.

Check your answers with those at the end of the unit.

The tangent curve

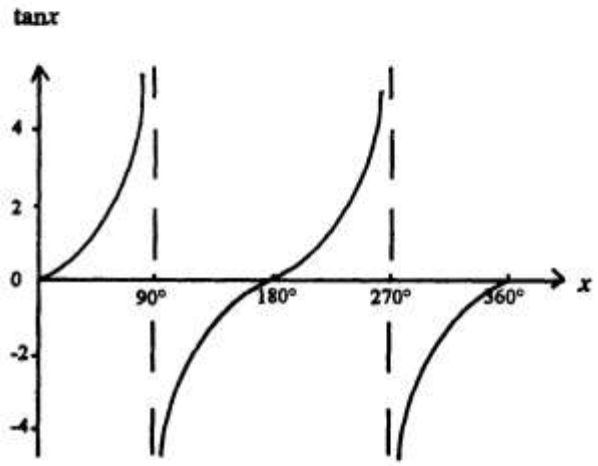
This curve is unlike the sine and cosine curves.

Try to find the value of tan 90° in your calculator. It should display E, for error. This is because the value is undefined. The calculator is unable to provide a value for tan 90° because tan 90° = ∞. (infinity). Infinity is an unmeasurable quantity and is very large.

We shall now proceed to make a table of values for tan x between 0° and 360°.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
tan x	0	0.58	1.73	∞	-1.73	-0.58	0	0.58	1.73	∞	-1.73	-0.58	0

The graph is drawn as shown below. Notice that the graph goes off to infinity at 90° and 270°. Dotted lines are drawn at these values. They are called asymptotes.



Try this last exercise.

Exercise E

1. Using values for x, from -180° to 540°, find the values of tan x in 30° intervals. Use these to draw a tangent curve on graph paper. Use a scale of 1cm. to represent 30° on the x axis and 1cm. to represent 1 unit on the y axis, mark the y axis between -4 and +4 as shown on the previous page. From the graph, write all the values of x, correct to the nearest degree for the following values of tan x.
   a) $\tan x = 1.0$
   b) $\tan x = 0.7$
   c) $\tan x = -0.7$
   d) $\tan x = 1.6$
   e) $\tan x = -1.6$
   f) $\tan x = 2.5$
   g) $\tan x = -3.25$
   h) $\tan x = -1.0$

Check your answers with those given at the end of this unit.

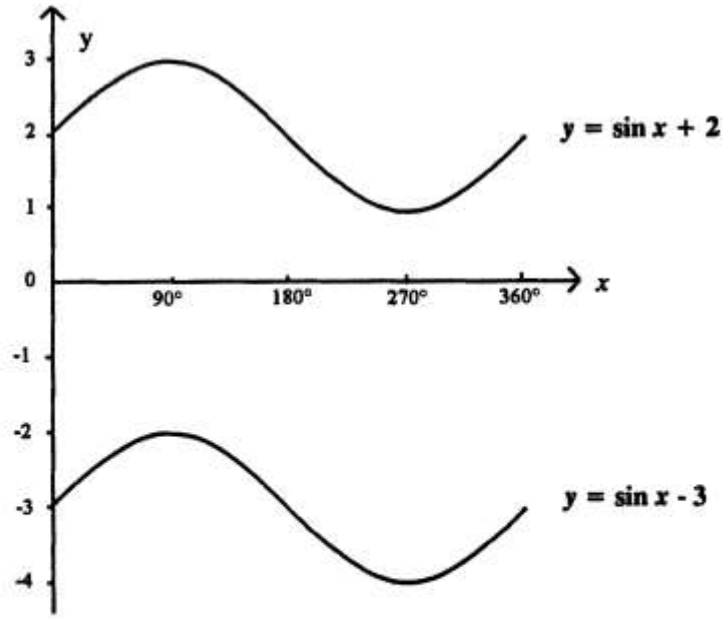
Answers

Exercise A

1. a) $x = 30°, 150°, 390°$ and $510°$
   b) $x = 45°, 135°, 405°$ and $495°$
   c) $x = -135°, -45°, 225°$ and $315°$
   d) $x = 53°, 127°, 413°$ and $487°$
   e) $x = -127°, -53°, 233°$ and $307°$
   f) $x = 64°, 116°, 424°$ and $476°$
   g) $x = -166°, -14°, 194°$ and $346°$
   h) $x = -150°, -30°, 210°$ and $330°$

Exercise B

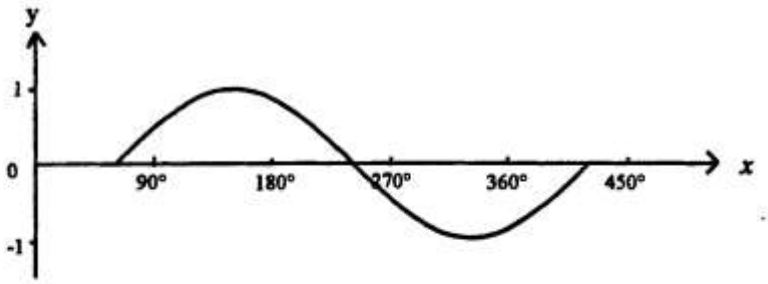
1.



The translation which tranposes $y = \sin x + 2$ onto $y = \sin x - 3$ is a translation of 5 points in the negative y direction.

Exercise C

1.

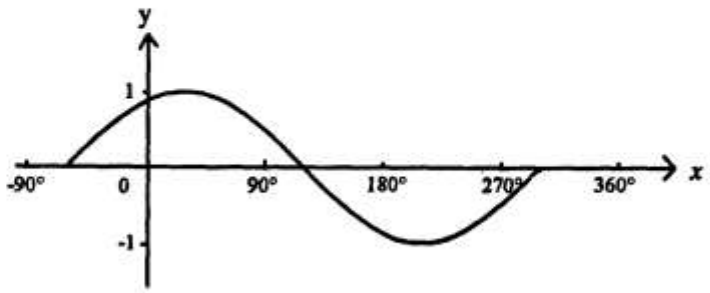


The translation which transposes $y = \sin x$ to $y = \sin(x - 60°)$ is a translation of $60°$ in the positive x direction.

Answers to Exercise C are continued on the next page.

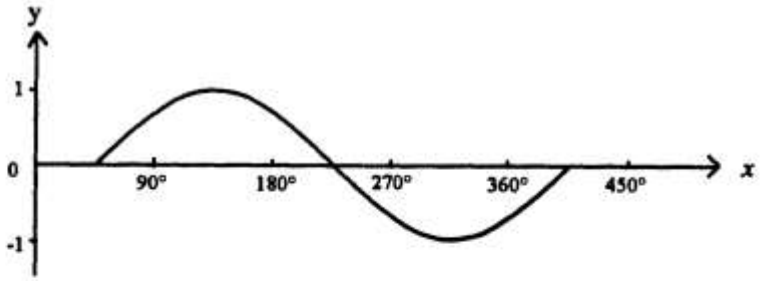
Exercise C (*continued*).

2.



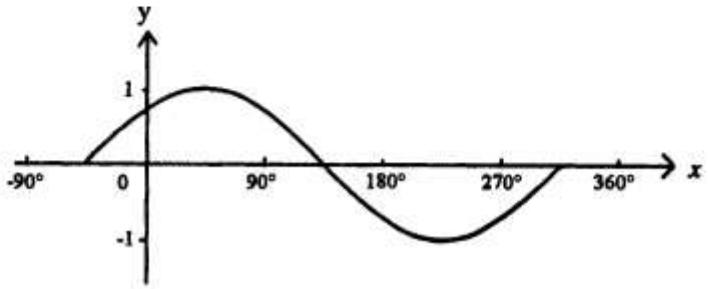
The translation which transposes $y = \sin x$ to $y = \sin(x + 60°)$ is a translation of 60° in the negative x direction.

3.



The translation which transposes $y = \sin x$ to $y = \sin(x - 45°)$ is a translation of 45° in the positive x direction.

4.



The translation which transposes $y = \sin x$ to $y = \sin(x + 45°)$ is a translation of 45° in the negative x direction.

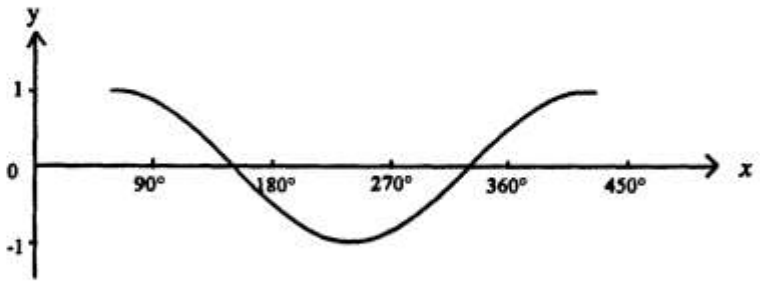
Exercise D

1. a) x = -60°, 60°, 300° and 420°
   b) x = -45°, 45°, 315° and 405°
   c) x = -135°, 135°, 225° and 495°
   d) x = -37°, 37°, 323° and 397°
   e) x = -143°, 143°, 217° and 503°
   f) x = -26°, 26°, 334° and 386°
   g) x = -104°, 104°, 256° and 464°
   h) x = -120°, 120°, 240° and 480°

Answers to Exercise D are continued on the next page.

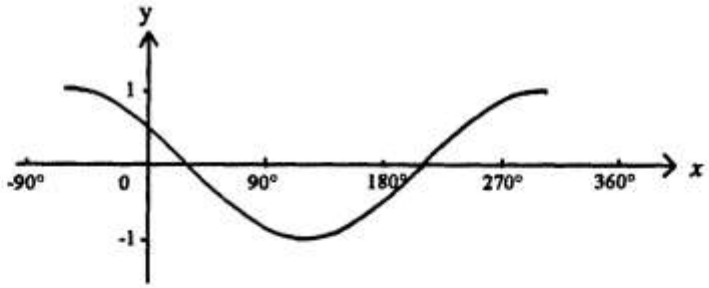
Exercise D (*continued*).

2.



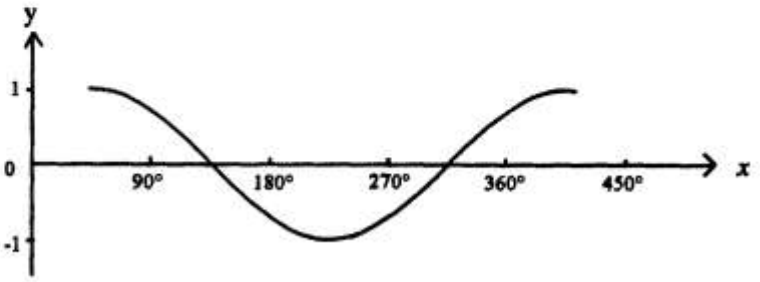
The translation which transposes $y = \cos x$ to $y = \cos(x - 60°)$ is a translation of 60° in the positive x direction.

3.



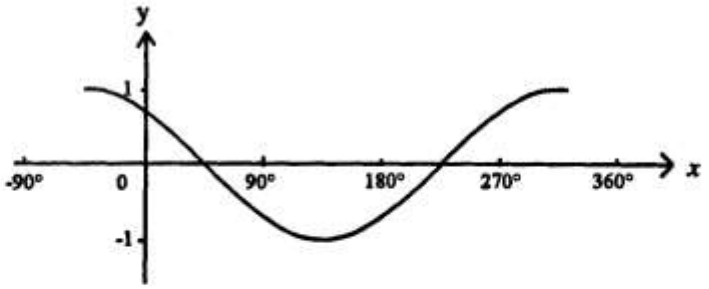
The translation which transposes $y = \cos x$ to $y = \cos(x + 60°)$ is a translation of 60° in the negative x direction.

4.



The translation which transposes $y = \cos x$ to $y = \cos(x - 45°)$ is a translation of 45° in the positive x direction.

5.

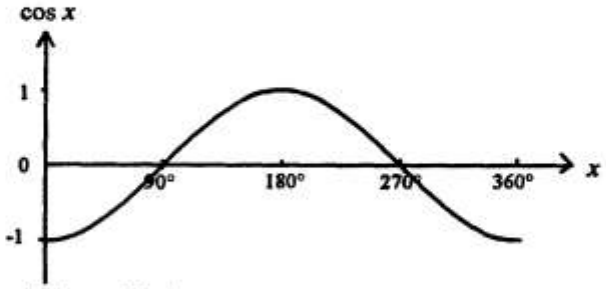


The translation which transposes $y = \cos x$ to $y = \cos(x + 45°)$ is a translation of 45° in the negative x direction.

Answers to Exercise D are continued on the next page.

Exercise D (*continued*).

6.



The translation which transposes $y = \cos x$ to $y = -\cos x$ is a translation of 180° in the positive or negative x direction or a reflection in the x axis.

Exercise E

1. a) x = -135°, 45°, 225° and 405°
   b) x = -145°, 35°, 215° and 395°
   c) x = -35°, 145°, 325° and 505°
   d) x = -122°, 58°, 238° and 418°
   e) x = -58°, 122°, 302° and 482°
   f) x = -112°, 68°, 248° and 428°
   g) x = -73°, 107°, 287° and 467°
   h) x = -45°, 135°, 315° and 495°