Unit 18

Sine, cosine and tangent ratios

Objectives

On completion of this unit you should be able to:

1. Use the sine, cosine and tangent ratios to find the lengths of sides and the angle of a triangle.
2. Use the inverse of the three trig ratios.
3. Find angles and sides of composite figures.

Cook, M. and Rimmer, H. (1993),
Stafford College, Earl Street, Stafford, ST16 2QR

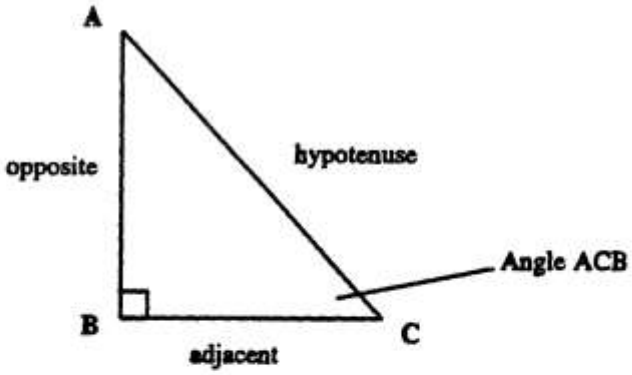
Trigonometry (Trig)

Trig is the study of the relationships between the angles and the sides in triangles.

The ratios used in this unit are only applicable to right angled triangles. Note that the diagrams in this unit are not drawn to scale, so you may not find any sides or angles by measurement.

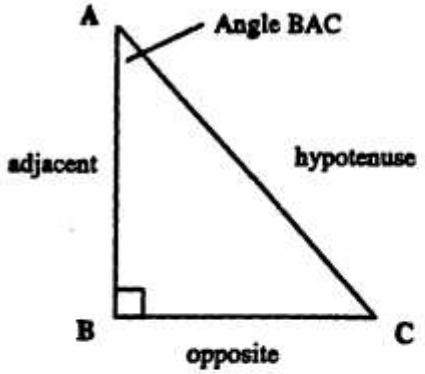
We shall name the sides of the right-angled triangle first.

Consider this triangle.



The **hypotenuse** is always the longest side. The longest side is the side opposite the right angle. In this case the hypotenuse is AC.
The side opposite the angle given, or required, is called the **opposite** side.
In the case of the triangle above, angle ACB is opposite side AB.
The other side is adjacent to the angle given or required and is called the **adjacent** side. BC is adjacent to angle ACB in the diagram above.

In the triangle below, the angle given, or required is the angle BAC.



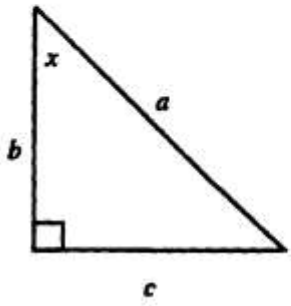
The sides are marked relative to the angle BAC this time.

Try this short exercise.

Exercise A

Name the sides relative to the marked angle in each case.

1.



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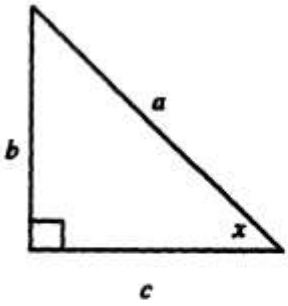

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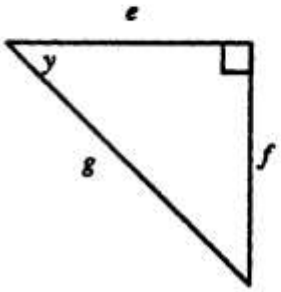

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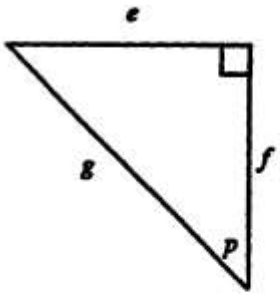


Check your answers with those at the end of the booklet.

Trig ratios

There are three trig ratios.
These ratios are sine (sin), cosine (cos) and tangent (tan).
The trig ratios are given below.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad S = \frac{O}{H}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad C = \frac{A}{H}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \qquad T = \frac{O}{A}$$

You may have heard these ratios referred to as,

SOH-CAH-TOA

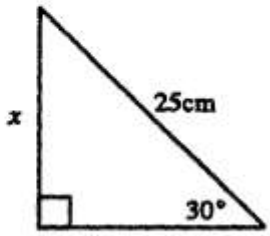
This should help you to remember the ratios.

The sin ratio

Study the following examples.

Example 1

Using the following triangle, find the size of the marked side.



The marked side, x, is opposite the angle 30°.
The hypotenuse is opposite the right angle, and is equal to 25cm.
The two sides involved are therefore the opposite side and the hypotenuse. O and H.

The letters O and H are together in SOH, so we can say that,

$$\sin 30° = \frac{O}{H} = \frac{x}{25}$$

Multiply both sides of the equation by 25.

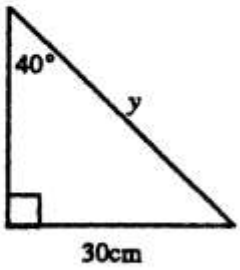
$$25 \times \sin 30° = x$$

You now need to use the button on your calculator marked **sin**

Calculator: 25 x 30 sin = 12.5

The length of x is 12.5cm.

Example 2
Find the value of y to one decimal place.



y is opposite the right angle, so y is the hypotenuse.
The side opposite the 40° angle is 30cm.
We need to use the sides O and H.
This is then SOH and we use the sin of angle 40° as follows,

$$\sin 40° = \frac{O}{H}$$

$$\sin 40° = \frac{30}{y}$$

Multiply both sides by y.

$$y \times \sin 40° = 30$$

We wish to find the value of y, y is multiplied by sin40°, so we divide both sides of the equation by sin40°.

$$y = \frac{30}{\sin 40°}$$

Calculator: 30 + 40 sin = 46.671715

To one decimal place y = 46.7cm.

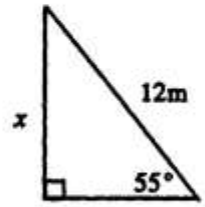
This is the hypotenuse of the triangle and is therefore the longest side of the triangle. Always check that your answers are sensible.

Now try the exercise on the next page.

Exercise B

Find the side marked x in each case, giving your answer correct to one decimal place.

1.



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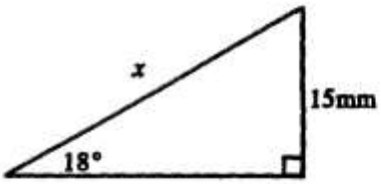

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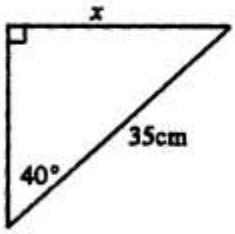

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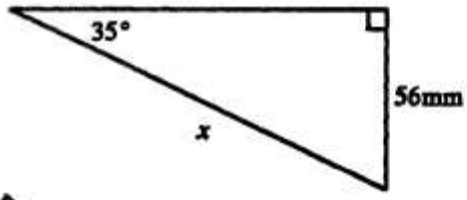

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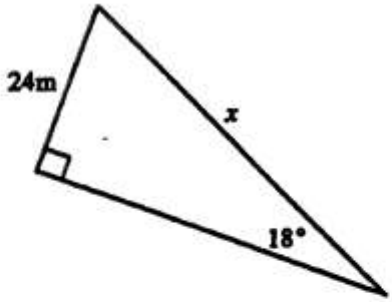


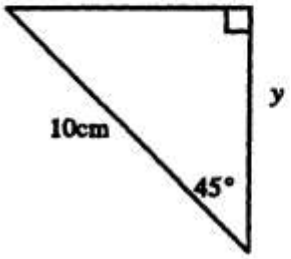
Check your answers with those at the end of the unit.

The cos ratio

Study these examples.

Example 3

Find the value of y in the following diagram. Give the answer to 2 decimal places.



The hypotenuse is the side opposite the right angle and is equal to 10cm.
y is the side adjacent to the angle of 45°.
We are going to use the sides A and H.
SOH-CAH-TOA
A and H are both in CAH.
We shall use the cosine of the angle.

$$\cos 45° = \frac{A}{H} = \frac{y}{10}$$

Multiply both sides by 10.

$$10 \times \cos 45° = y$$

$$y = 10 \times \cos 45°$$

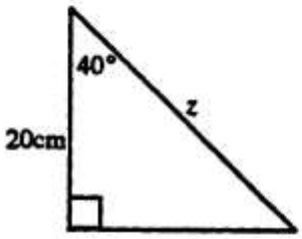
Find the **cos** button on your calculator.

Calculator: 10 x 45 cos = 7.0710678

The length of y is 7.07cm. to 2 decimal places.

Example 4

Find the value of z, to one decimal place.



As before check to find which sides are involved.
z is the side opposite the right angle,
so it is the hypotenuse.
The 20cm. side is adjacent to the angle of 40°.
We need to use A and H, CAH.

$$\cos 40° = \frac{A}{H} = \frac{20}{z}$$

Multiply both sides by z.

$$z \times \cos 40° = 20$$

Divide both sides by cos40°.

$$z = \frac{20}{\cos 40°}$$

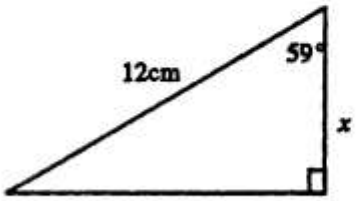
z = 26.1cm. to one decimal place.

Try this exercise.

Exercise C

Find the side marked x in the following diagrams, giving your answers correct to one decimal place.

1.



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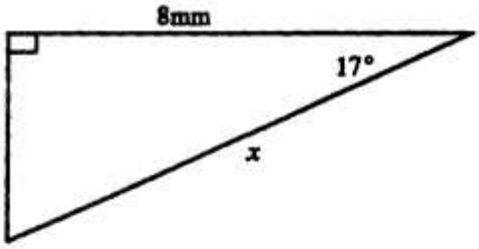

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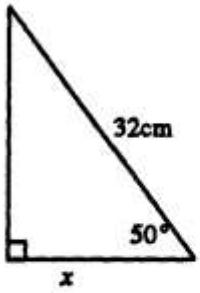

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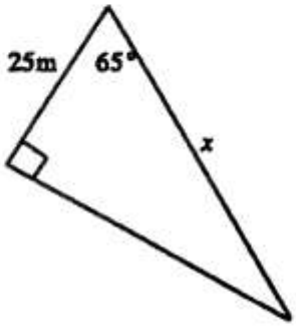

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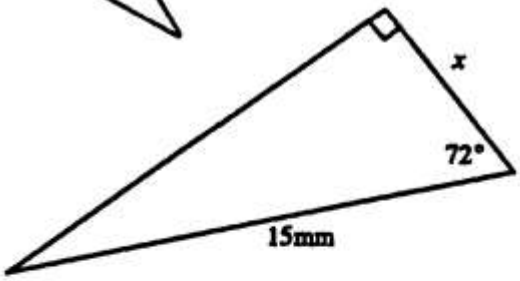
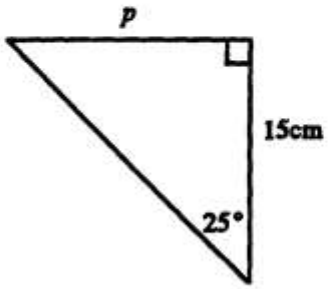


Check your answers with those at the end of the unit, before studying the next examples.

The tan ratio

Example 5
Find the value of p to one decimal place.



p is opposite the 25° angle.
The 15cm. side is adjacent to the 25° angle.
We need to use O and A, TOA.

$$\tan 25° = \frac{O}{A} = \frac{p}{15}$$

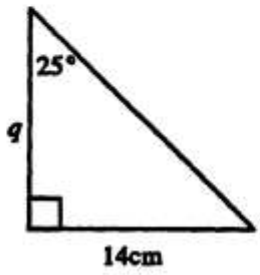
Multiply both sides by 15.

$$15 \times \tan 25° = p$$

Calculator: 15 x 25 tan = 6.9946149

There is a 9 in the first decimal place. There is also a nine in the second decimal place, so we need to round up. When we round up the first decimal place, $6.9 + 0.1 = 7.0$
p = 7.0cm. to one decimal place.

Example 6
Find the value of q to one decimal place.



The 14cm. side is opposite the angle 25°.
q is adjacent to the 25° angle.
O and A. Use TOA.

$$\tan 25° = \frac{O}{A}$$

$$\tan 25° = \frac{14}{q}$$

Multiply both sides by q.

$$q \times \tan 25° = 14$$

Divide both sides by tan25°.

$$q = \frac{14}{\tan 25°}$$

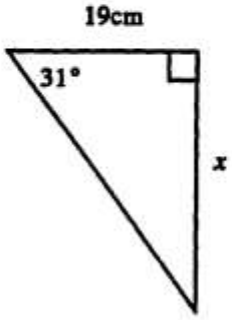
Using the calculator, q = 30.0cm. to one decimal place.

Try the exercise on the next page.

Exercise D

Find the value of x in each case, giving your answers correct to two decimal places.

1.



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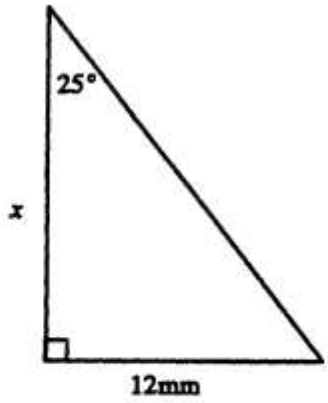

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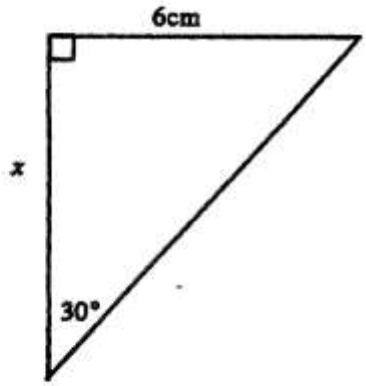

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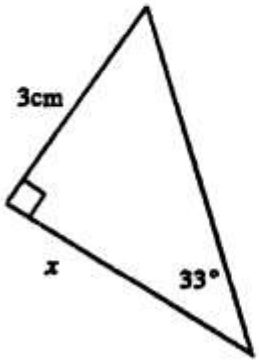

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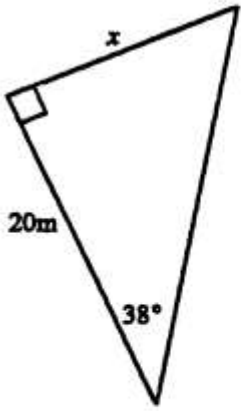

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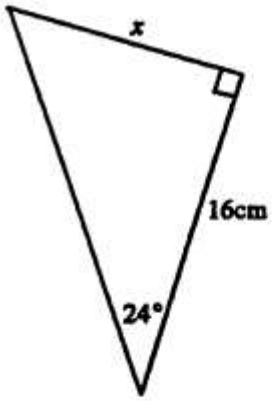


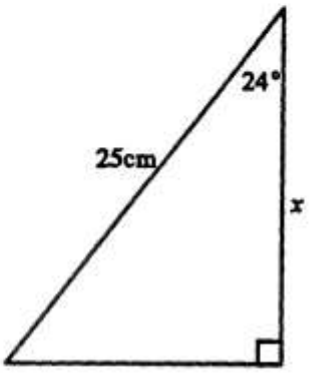
Check your answers with those at the end of the unit.

Try this miscellaneous exercise.

Exercise E

Find the length of the side marked x in the following exercise, giving your answers correct to two decimal places. You will need to decide whether to use sin, cos or tan.

1.



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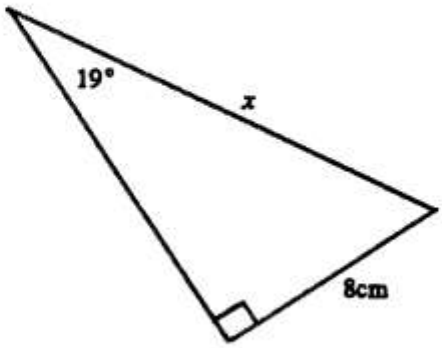

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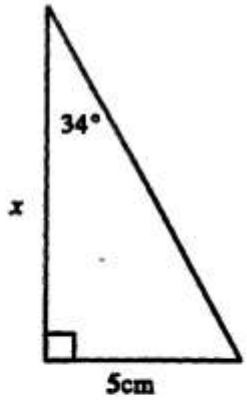

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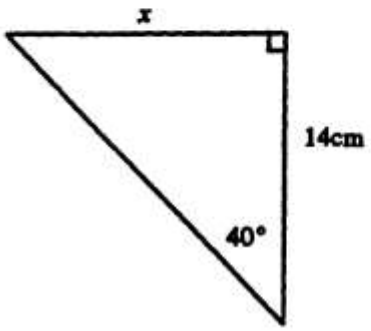

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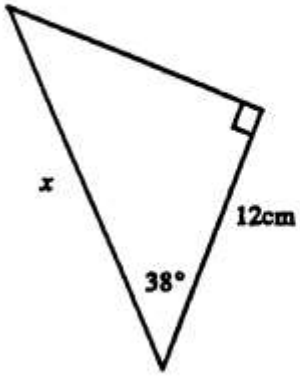

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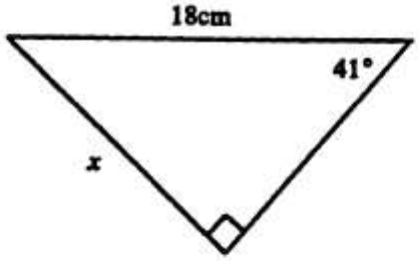


Check your answers with those at the end of the unit.

Use of the inverse trig ratios

So far we have found the lengths of sides given the length of one side and an angle.
Sometimes you will be given two sides of a right angled triangle and asked to calculate an angle.
Note that the diagrams in this section are not drawn to scale, so you may not find any sides or angles by measurement.

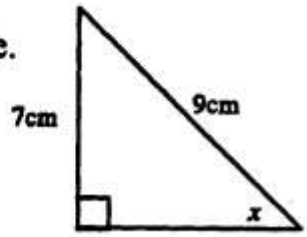
Consider the following examples.

Example 7
Given that sin A = 0.5, find the value of angle A.

You need to use the **INV** button on your calculator. This may be the 2nd function button on some calculators.

Calculator 0.5 INV sin 30°
Angle A is 30°.

Example 8
Find the value of x to the nearest degree.



The hypotenuse is 9cm.
The 7cm. side is opposite the required angle.

Sides O and H are involved. SOH. We need to use the sin of angle x.

$$\sin x = \frac{O}{H}$$

$$\sin x = \frac{7}{9}$$

Calculator: 7 ÷ 9 = INV sin 51.057559

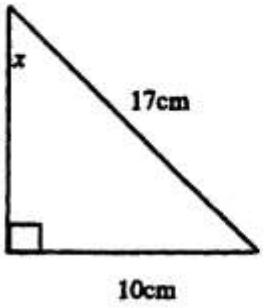
x is 51°.

Try this exercise.

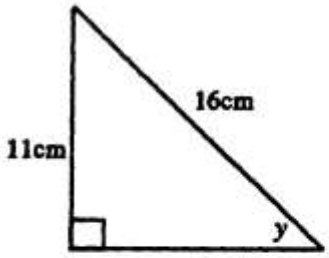
Exercise F

Give all your answers to the nearest degree.

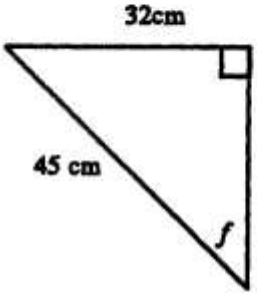
1. Find angle x.



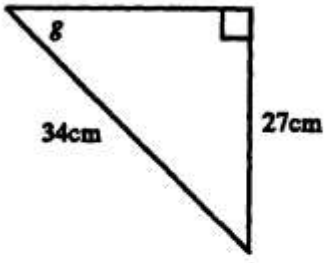
2. Find angle y.



3. Find angle f.

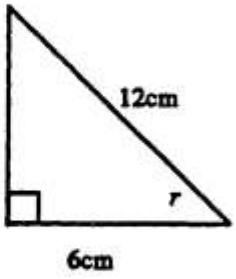


4. Find angle g.



Check your answers.

Example 9
Find the value of r.



The hypotenuse is 12cm.
The side adjacent to the required angle is 6cm.
A and H. CAH.
Use the cos of the angle.

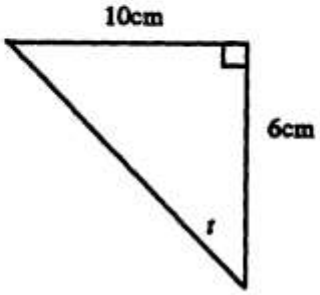
$$\cos r = \frac{A}{H}$$

$$\cos r = \frac{6}{12}$$

Calculator: 6 ÷ 12 = INV cos 60

Angle r is equal to 60°.

Example 10
Find the value of t to the nearest degree.



The side opposite t is 10cm.
The side adjacent to t is 6cm.
O and A. TOA. Use the tan of the angle.

$$\tan t = \frac{O}{A}$$

$$\tan t = \frac{10}{6}$$

Calculator: 10 ÷ 6 = INV tan 59.036243

t is equal to 59°.

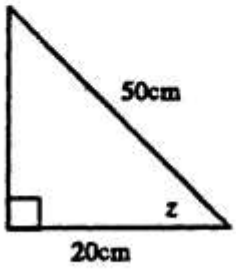
Try the exercise on the next page.

Try this exercise.

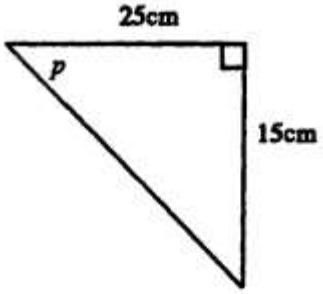
Exercise G

Give all answers in this exercise correct to one decimal place.

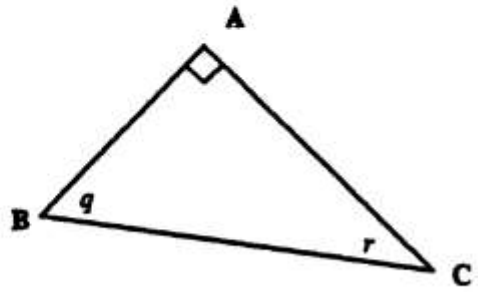
1. Find angle z.



2. Find angle p.



3. Using the following triangle complete a) to h).



You will find it easier if you draw the relevant triangle each time.

	AB	AC	BC	
a)	8cm.		12cm.	Find q.
b)		13cm.	20cm.	Find r.
c)	5.5cm.	3.2cm.		Find r.
d)		7cm.	15cm.	Find q.
e)	6.4cm.		8.2cm.	Find r.
f)	25cm.	56cm.		Find q.
g)	6.5cm.		10.5cm.	Find q and r.
h)	4.5cm.	6cm.	7.5cm.	Find q and r.

Check your answers, before continuing.

Composite figures

Trig questions may involve more than one triangle. Each triangle needs to be considered independently when deciding which side is the hypotenuse, the opposite and the adjacent.

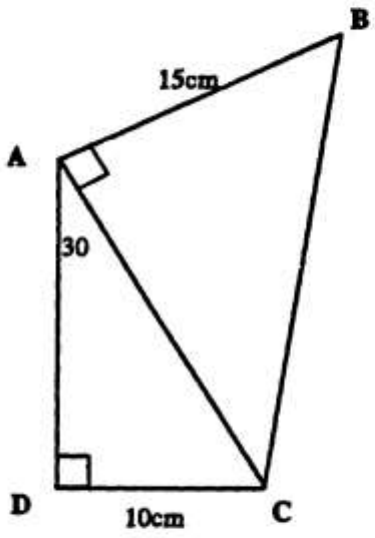
Consider these examples.

Example 11

Using the diagram shown,

a) find the length of AC,

b) calculate angle BCA to the nearest degree.



a) Using triangle ADC,
AC is the hypotenuse,
DC is opposite the angle 30°.
Use O and H. Use SOH.

$$\sin 30° = \frac{10}{AC}$$

Multiply both sides by AC.

$$AC \times \sin 30° = 10$$

Divide both sides by sin30°.

$$AC = \frac{10}{\sin 30°}$$

Using your calculator, you should find that AC = 20cm.

b) We now use triangle ABC.
AC, which we have just found is adjacent to the required angle, BCA.
AB is opposite angle BCA.
We need to use O and A. Use TOA.

$$\tan BCA = \frac{15}{20}$$

Calculator: 15 ÷ 20 = INV tan 36.869898

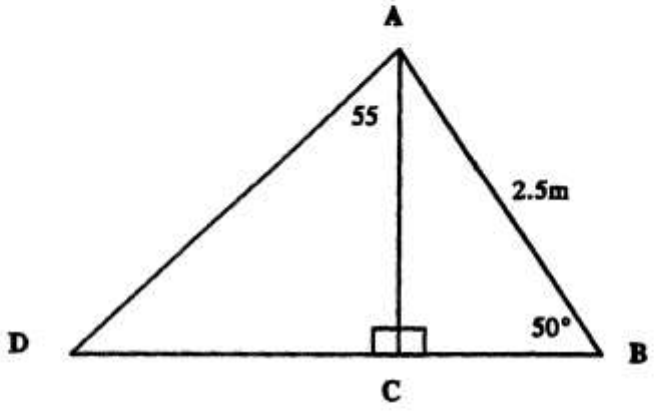
To the nearest degree, angle BCA is equal to 37°.

Example 12

The diagram shows the sail of a ship. AC is the mast. ACB and ACD are two sails. AB = 2.5m. Find, to three significant figures,

a) the height of the mast, AC,

b) the length BD.



a) Using the triangle ABC, AB is the hypotenuse, AC is opposite the angle 50°. We need to use O and H. Use SOH.

$$\sin 50° = \frac{AC}{2.5}$$

Multiply both sides by 2.5.

$$2.5 \times \sin 50° = AC$$

$$AC = 1.9151111\text{m.}$$

AC is equal to 1.92m. to three significant figures.

b) To find BD, we need to find BC and CD.

First BC. This is adjacent to the 50° angle. We shall use A and H. Use CAH.

$$\cos 50° = \frac{BC}{2.5}$$

Multiply both sides by 2.5.

$$BC = 2.5 \times \cos 50° = 1.606969\text{m.}$$

Now we shall find CD, using the triangle ACD.

CD is opposite the 55° angle. AC is adjacent to the 55° angle. Use O and A. Use TOA. Use AC = 1.9151111 by leaving it in the memory of your calculator. This will make your final answer more accurate.

$$\tan 55° = \frac{CD}{1.9151111}$$

Multiply both sides by 1.9151111.

$$CD = 1.9151111 \times \tan 55° = 2.7350621\text{m.}$$

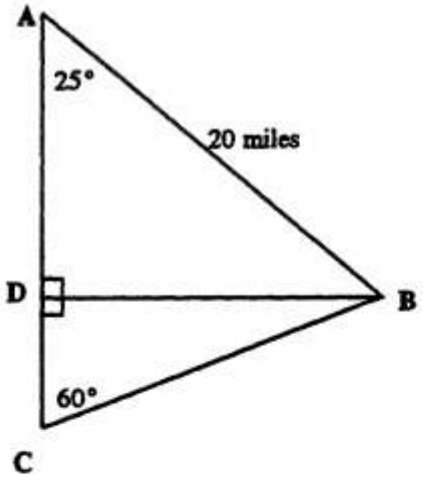
$$BD = 1.606969 + 2.7350621 = 4.3420311\text{m.}$$

BD is equal to 4.34m. to three significant figures.

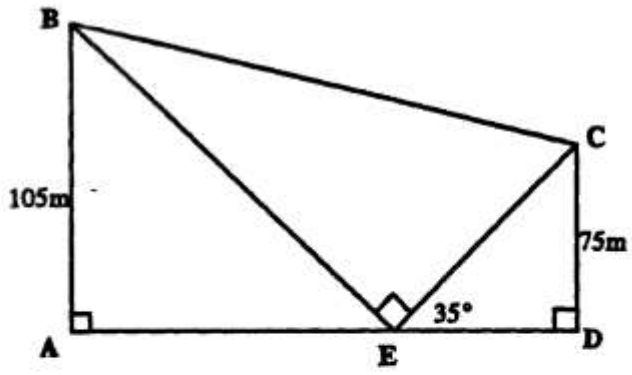
Try this exercise.

Exercise H

1. The diagram below is of a motorway system linking four towns marked A, B, C and D. The distance between A and B is 20 miles, angle DAB = 25° and angle DCB = 60°. Calculate correct to one decimal place,
   a) the distance AD,
   b) the distance BD,
   c) the distance DC,
   d) the total distance AC.



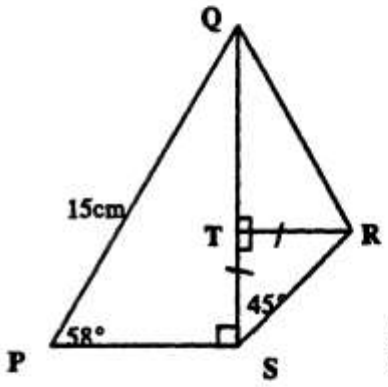
2. The diagram below shows two pylons marked AB and CD, where AB is of height 105m. and CD is of height 75m. Angle CED is 35° and AED is a straight line. Find, correct to the nearest whole number,
   a) the length of CE,
   b) the size of angle AEB,
   c) the length of BE,
   d) the size of angle ECB.



Exercise H is continued on the next page.

Exercise H (*Continued*)

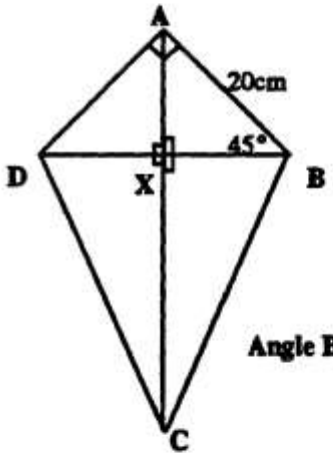
3. In the diagram shown below PQ = 15cm. and angle SPQ = 58°. Triangle STR is isosceles with angle TSR = 45° and ST = TR = 5.5cm. Calculate, correct to one decimal place,
   a) the length of QS,
   b) the length of QT,
   c) the size of angle QRT to the nearest degree.



Angle TSR = Angle TRS = 45°
ST = TR = 5.5cm

4. The diagram below is of a child's kite in which AC is a line of symmetry. If AB is 20cm. and angle BCD is 32°. Find,
   a) the length of AX,
   b) the length of BX,
   c) the length of CX,
   d) the total length of the kite, AC.

Give all your answers correct to the nearest whole number.



Angle BCD = 32°

Check your answers with those at the end of the unit.

Answers

Exercise A

1. a = hypotenuse
   b = adjacent
   c = opposite
2. a = hypotenuse
   b = opposite
   c = adjacent
3. e = adjacent
   f = opposite
   g = hypotenuse
4. e = opposite
   f = adjacent
   g = hypotenuse

Exercise B

1. 9.8m.
2. 48.5mm.
3. 22.5cm.
4. 97.6mm.
5. 77.7m.

Exercise C

1. 6.2cm.
2. 8.4mm.
3. 20.6cm.
4. 59.2m.
5. 4.6mm.

Exercise D

1. 11.42cm.
2. 25.73mm.
3. 10.39cm.
4. 4.62cm.
5. 15.63m.
6. 7.12cm.

Exercise E

1. 22.84cm.
2. 24.57cm.
3. 7.41cm.
4. 11.75cm.
5. 15.23cm.
6. 11.81cm.

Exercise F

1. 36°
2. 43°
3. 45°
4. 53°

Exercise G

1. 66.4°
2. 31.0°
3. a) $q = 48.2°$
   b) $r = 49.5°$
   c) $r = 59.8°$
   d) $q = 27.8°$
   e) $r = 51.3°$
   f) $q = 65.9°$
   g) $q = 51.8°$ $r = 38.2°$
   h) $q = 53.1°$ $r = 36.9°$

Exercise H

1. a) 18.1miles
   b) 8.5 miles
   c) 4.9 miles
   d) 23.0 miles
2. a) 131m.
   b) 55°
   c) 128m.
   d) 44°
3. a) 12.7cm.
   b) 7.2cm.
   c) 53°
4. a) 14cm.
   b) 14cm.
   c) 49cm.
   d) 63cm.