MATHS Xtra ALGEBRA

Unit 16

Matrices

Objectives

On completion of this unit you should be able to:

1. Form and transpose matrices.

2. Multiply matrices.

3. Find the determinant and inverse of a matrix.

4. Solve simultaneous equations using matrices.

5. Solve equations using matrices.

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Matrices

A matrix is a means of organising information.
The order of a matrix is given by,

number of rows x number of columns.

Study this example.

Example 1

Write a matrix for the following information.
Mary, Jane, Edward and Jarved obtained the following marks in English, Maths and Science,

	English	Maths	Science
Mary	32	45	67
Jane	45	33	62
Edward	43	31	68
Jarved	48	44	61

This can be displayed by a 4 x 3 matrix.

$$\begin{pmatrix} 32 & 45 & 67 \\ 45 & 33 & 62 \\ 43 & 31 & 68 \\ 48 & 44 & 61 \end{pmatrix}$$

Note
This matrix has 4 rows and 3 columns.

The information could also have been displayed as,

	Mary	Jane	Edward	Jarved
English	32	45	43	48
Maths	45	33	31	44
Science	67	62	68	61

This gives the 3 x 4 matrix,

$$\begin{pmatrix} 32 & 45 & 43 & 48 \\ 45 & 33 & 31 & 44 \\ 67 & 62 & 68 & 61 \end{pmatrix}$$

Note
This matrix has 3 rows and 4 columns.

Either matrix would be a correct solution. There are several solutions to this question depending on the order in which the columns and rows are labelled.

Try the short exercise on the next page.

Exercise A

Using the information in each question and the suggested headings, write an appropriate matrix. State the order of the matrix in each case.

1. A pet food manufacturer has two types of cat food, Meow and Purrs. He sells 5 cases of Meow to shop A and 8 cases to shop B. Shop A also purchases 7 cases of Purrs, while shop B purchases 3 of these. Shop C buys 9 cases of Meow and 6 cases of Purrs.

	Shop A	Shop B	Shop C
Meow			
Purrs			

2. Two football teams, team A and team B had the following results after five games. Team A had won 1 game, drawn 2 games and lost 2 games. Team B had won 3, and drawn 2 games.

	Team A	Team B
Won		
Drawn		
Lost		

3. A group of young people were asked to select their favourite sport from a list of, football, netball, swimming, tennis and golf. Of the group of girls, 12 chose tennis, 8 chose swimming, 13 chose netball and 2 chose golf. Of the group of boys, 18 chose football, 8 chose tennis, 7 chose swimming and 3 chose golf.

	Football	Netball	Swimming	Tennis	Golf
Girls					
Boys					

4. There are four houses in a school, Alpha, Beta, Gamma and Delta. In Alpha there are 185 girls and 215 boys, in Beta there are 177 girls and 191 boys, in Gamma there are 202 girls and 176 boys and in Delta there are 206 girls and 184 boys.

	Alpha	Beta	Gamma	Delta
Girls				
Boys				

5. In a questionnaire 10 people from each of three age ranges were questioned about their pets. In the 0 to 10 age group, 6 had dogs, 2 had cats and 2 had fish. In the 11 to 20 age group, 2 had dogs, 3 had cats, 3 had fish, 1 had mice and 1 had rabbits. In the 21 to 30 age range, 2 had birds, 4 had dogs, 1 had a rabbit, 1 had fish and 2 had cats.

	Dogs	Cats	Rabbits	Mice	Fish	Birds
0 - 10						
11 - 20						
21 - 30						

Check your answers with those at the end of the booklet.

Transposing matrices

In Example 1, a 4 x 3 matrix was also displayed as a 3 x 4 matrix. We say that the 4 x 3 matrix was transposed into a 3 x 4 matrix. The transpose of a matrix is written as A' or A^T. The first row becomes the first column, the second row becomes the second column and so on.

Study this example.

Example 2

a) If $A = \begin{pmatrix} 2 & -5 & 7 \\ 1 & 0 & 6 \end{pmatrix}$ write the matrix A^T.

b) If $B = \begin{pmatrix} 1 & 0 \\ 7 & 3 \end{pmatrix}$ write the matrix B'.

a) A is a 2 x 3 matrix. We can display it as a 3 x 2 matrix as follows.

$$A^T = \begin{pmatrix} 2 & 1 \\ -5 & 0 \\ 7 & 6 \end{pmatrix}$$

b) This 2 x 2 matrix is transposed as follows.

$$B' = \begin{pmatrix} 1 & 7 \\ 0 & 3 \end{pmatrix}$$

Try this short exercise.

Exercise B

1. If $A = \begin{pmatrix} -2 & 32 & -5 & 7 \\ 21 & 0 & 6 & 8 \end{pmatrix}$ write the matrix A^T.
2. If $B = \begin{pmatrix} 1 & 0 & -5 \\ 7 & 3 & 1 \end{pmatrix}$ write the matrix B'.
3. If $C = \begin{pmatrix} -2 & 36 & 15 & 7 \\ 14 & 0 & 3 & 24 \\ 10 & 9 & 12 & 11 \end{pmatrix}$ write the matrix C^T.
4. If $D = \begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}$ write the matrix D'.
5. If $E = \begin{pmatrix} 12 & -5 & -2 \\ 21 & 0 & -6 \end{pmatrix}$ write the matrix E^T.
6. If $F = \begin{pmatrix} -2 & 27 \\ 21 & 0 \\ 10 & 9 \\ 1 & -1 \end{pmatrix}$ write the matrix F^T.

Check your answers with those at the end of the unit.

Using matrices

Study this example.

Example 3

A nursery supplies garden centres with three types of plants, as shown in the table below.

	Centre A	Centre B	Centre C
fuchsia	10 boxes	12 boxes	14 boxes
geranium	8 boxes	7 boxes	6 boxes
lobelia	15 boxes	20 boxes	6 boxes

a) Display this information as a 3 x 3 matrix.

b) If the cost of the boxes of plants is as follows,
fuchsia £2 geranium £3 lobelia £1
calculate the total cost to each garden centre.

a) $$\begin{pmatrix} 10 & 12 & 14 \\ 8 & 7 & 6 \\ 15 & 20 & 6 \end{pmatrix}$$

b) We now calculate the cost by multiplying, **row x column.**

$$(2 \quad 3 \quad 1)\begin{pmatrix} 10 & 12 & 14 \\ 8 & 7 & 6 \\ 15 & 20 & 6 \end{pmatrix} = \begin{pmatrix} 2 \times 10 = 20 & 2 \times 12 = 24 & 2 \times 14 = 28 \\ 3 \times 8 = 24 & 3 \times 7 = 21 & 3 \times 6 = 18 \\ 1 \times 15 = 15 & 1 \times 20 = 20 & 1 \times 6 = 6 \end{pmatrix}$$

The total to each centre can be found,

$$(20 + 24 + 15 \quad 24 + 21 + 20 \quad 28 + 18 + 6)$$
$$= (59 \quad 65 \quad 52)$$

We have combined a 1 x 3 matrix with a 3 x 3 matrix and produced a matrix of the order 1 x 3.

The total costs are £59 for Centre A, £65 for Centre B and £52 for Centre C.

Example 4

Combine the following row and column matrices.

$$(2 \quad 3)\begin{pmatrix} 1 & 4 & 6 \\ 7 & 8 & 5 \end{pmatrix}$$

We multiply **row x column**.

$$\begin{pmatrix} 2 \times 1 = 2 & 2 \times 4 = 8 & 2 \times 6 = 12 \\ 3 \times 7 = 21 & 3 \times 8 = 24 & 3 \times 5 = 15 \end{pmatrix} = (2 + 21 \quad 8 + 24 \quad 12 + 15)$$
$$= (23 \quad 32 \quad 27)$$

Note that we have combined a 1 x 2 matrix with a 2 x 3 matrix to produce a 1 x 3 matrix.

Before attempting the following questions you should notice what happens when two matrices are combined.

Combining,

1 x 3 with a 3 x **2** produces a **1** x **2**,
2 x 4 with a 4 x **3** produces a **2** x **3**,
3 x 2 with a 2 x **4** produces a **3** x **4**.

Notice also that the number of columns of the first matrix must be the same as the number of rows of the second matrix for it to be possible to combine the matrices.

Try the following exercise.

Exercise C

Combine the following row and column matrices.

1. $\begin{pmatrix} 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & 4 \end{pmatrix}$

2. $\begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 & 1 \\ 1 & 0 & 6 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 5 & 8 & 2 \end{pmatrix} \begin{pmatrix} 7 & 12 \\ 3 & 9 \\ 1 & 4 \end{pmatrix}$

4.a) Form a matrix for the following results for three football teams.

Team A	Team B	Team C
4 wins	3 wins	6 wins
6 draws	7 draws	3 draws
2 losses	2 losses	3 losses

b) If a win is worth 2 points, a draw is worth 1 point and there are 0 points for a loss, combine two matrices to calculate the total points for each team.

5.a) A newsagent kept a record of the sales of the two most popular magazines over a four week period. Magazine A sold 24, 16, 17 and 21 copies in each of the weeks. Magazine B sold 15, 18, 28 and 23 copies in each of the weeks. Form a matrix to display this information.

b) If magazine A costs £1.25 and magazine B costs £1.20, combine two matrices to find the total for the sales of magazines A and B on each of the weeks recorded.

Check your answers with those at the end of the unit.

Multiplying matrices

Consider this example.

Example 5

$$P = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 8 & 7 \end{pmatrix} \qquad Q = \begin{pmatrix} 3 & -4 & 0 \\ -1 & -2 & -3 \end{pmatrix}$$

Find,

a) PQ,
b) QP.

a) $$\begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 8 & 7 \end{pmatrix}\begin{pmatrix} 3 & -4 & 0 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 1 \times 3 + 2 \times -1 & 1 \times -4 + 2 \times -2 & 1 \times 0 + 2 \times -3 \\ 5 \times 3 + 6 \times -1 & 5 \times -4 + 6 \times -2 & 5 \times 0 + 6 \times -3 \\ 8 \times 3 + 7 \times -1 & 8 \times -4 + 7 \times -2 & 8 \times 0 + 7 \times -3 \end{pmatrix}$$

$$PQ = \begin{pmatrix} 1 & -8 & -6 \\ 9 & -32 & -18 \\ 17 & -46 & -21 \end{pmatrix}$$

Notice that by multiplying a 3 x 2 matrix by a 2 x 3 matrix we have produced a 3 x 3 matrix.

b) $$\begin{pmatrix} 3 & -4 & 0 \\ -1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 8 & 7 \end{pmatrix} = \begin{pmatrix} 3 \times 1 + -4 \times 5 + 0 \times 8 & 3 \times 2 + -4 \times 6 + 0 \times 7 \\ -1 \times 1 + -2 \times 5 + -3 \times 8 & -1 \times 2 + -2 \times 6 + -3 \times 7 \end{pmatrix}$$

$$QP = \begin{pmatrix} -17 & -18 \\ -35 & -35 \end{pmatrix}$$

We have multiplied a 2 x 3 matrix by a 3 x 2 matrix and obtained a 2 x 2 matrix.

You can see that PQ is **not** the same as QP.

Turn to the next page to try the next exercise.

Exercise D

If,

$A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}$ $D = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & -1 \\ 3 & 2 & -5 \end{pmatrix}$

$E = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ $F = \begin{pmatrix} 3 & 6 \\ 1 & 0 \\ -2 & 4 \end{pmatrix}$ $G = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 0 & -3 \end{pmatrix}$ $H = \begin{pmatrix} 1 & -2 & 1 \\ 3 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix}$

Find, where possible,

1. AB,
2. BC,
3. A^2 Hint, write out AA.
4. DC
5. DB
6. EG
7. GE
8. FE
9. GH
10. H^2

Check your answers with those at the end of the unit.

The identity matrix

If the matrix A is multiplied by the identity matrix then the result is matrix A. An identity matrix is usually denoted by the letter I. This is a 2 x 2 identity matrix, where,

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

For example,

if $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ then, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.

The zero or null matrix

The zero or null matrix is usually denoted by O, where,

$$O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

is a null 2 x 2 matrix. Any matrix multiplied by O, will produce the matrix O, as follows.

If,

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \quad \text{then} \quad OA = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Multiplication by a scalar

A matrix may be multiplied by a number (scalar) as shown in the next example.

Example 6

a) Simplify $3\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}$.

b) Simplify $-\frac{1}{2}\begin{pmatrix} 3 & -2 \\ -1 & 0 \end{pmatrix}$.

c) If, $A = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 1 \\ 0 & 6 \end{pmatrix}$

write the matrix $2A + B^T$.

a) $$3\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 3 \times 1 & 3 \times 3 \\ 3 \times 2 & 3 \times (-5) \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 6 & -15 \end{pmatrix}.$$

b) $$-\frac{1}{2}\begin{pmatrix} 3 & -2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1/2 \times 3 & -1/2 \times (-2) \\ -1/2 \times (-1) & -1/2 \times 0 \end{pmatrix} = \begin{pmatrix} -1.5 & 1 \\ 0.5 & 0 \end{pmatrix}.$$

c) $$2A = \begin{pmatrix} 2 & 8 \\ -4 & -6 \end{pmatrix} \qquad B^T = \begin{pmatrix} 5 & 0 \\ 1 & 6 \end{pmatrix}$$

$$2A + B^T = \begin{pmatrix} 2+5 & 8+0 \\ -4+1 & -6+6 \end{pmatrix} = \begin{pmatrix} 7 & 8 \\ -3 & 0 \end{pmatrix}.$$

Try the exercise on the next page.

Exercise E

Simplify the following.

1. $2\begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$

2. $-1\begin{pmatrix} 5 & 1 \\ 0 & -1 \end{pmatrix}$

3. $-\frac{1}{2}\begin{pmatrix} -3 & 1 \\ 22 & -1 \end{pmatrix}$

4. $-2\begin{pmatrix} 3 & 1 \\ 0.5 & -2 \end{pmatrix}$

5. $\frac{1}{3}\begin{pmatrix} 3 & -12 \\ 6 & -3 \end{pmatrix}$

6. $2\begin{pmatrix} -3 & 0 \\ 0.5 & -4 \end{pmatrix}$

If $A = \begin{pmatrix} -6 & 2 \\ -1 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 0 & -4 \\ -2 & 10 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & -3 \\ 0 & 4 \end{pmatrix}$

Write the matrix represented by each of the following.

7. $2A + C$
8. $A^T + B^T$
9. $A^T + 2B - C$
10. $3B - C^T$

Check your answers with those at the end of the unit.

Determinant of a 2 x 2 matrix

If A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then its determinant is denoted by $|A|$ or det A.

where,

$|A| = ad - bc.$

Consider this example.

Example 7

If A = $\begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}$ find $|A|$.

$|A| = 2 \times (-1) - 3 \times 5 = (-2) - 15 = -17.$

Try this exercise.

Exercise F

1. If B = $\begin{pmatrix} 0 & 3 \\ 5 & -1 \end{pmatrix}$ find $|B|$.
2. If C = $\begin{pmatrix} 1 & -3 \\ 7 & 6 \end{pmatrix}$ find $|C|$.
3. If D = $\begin{pmatrix} -2 & 2 \\ 8 & -1 \end{pmatrix}$ find $|D|$.
4. If E = $\begin{pmatrix} -2 & 4 \\ 2 & 1 \end{pmatrix}$ find $|E|$.
5. If F = $\begin{pmatrix} -5 & -3 \\ 5 & -2 \end{pmatrix}$ find $|F|$.

Check your answers with those at the end of the unit.

The inverse of a 2 x 2 matrix

If A is a matrix, then its inverse matrix is denoted by A^{-1} and

$$AA^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

If, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Consider the example on the next page.

Example 8

a) If $A = \begin{pmatrix} 3 & -5 \\ 2 & -1 \end{pmatrix}$, find A^{-1}.

b) Find AA^{-1}.

a) We need to find $|A|$ first.
$|A| = [3 \times (-1) \; - \; (-5) \times 2] = 7$

so, $A^{-1} = \frac{1}{|A|}\begin{pmatrix} -1 & 5 \\ -2 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 5 \\ -2 & 3 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} {}^{-1}/_7 & {}^{5}/_7 \\ {}^{-2}/_7 & {}^{3}/_7 \end{pmatrix}$.

b) $AA^{-1} = \begin{pmatrix} 3 & -5 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} {}^{-1}/_7 & {}^{5}/_7 \\ {}^{-2}/_7 & {}^{3}/_7 \end{pmatrix}$

$= \begin{pmatrix} 3 \times {}^{-1}/_7 + (-5) \times {}^{-2}/_7 & 3 \times {}^{5}/_7 + (-5) \times {}^{3}/_7 \\ 2 \times {}^{-1}/_7 + (-1) \times {}^{-2}/_7 & 2 \times {}^{5}/_7 + (-1) \times {}^{3}/_7 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Try the following exercise.

Exercise G

Find the inverse of each of the following matrices.

1. $\begin{pmatrix} 1 & 5 \\ 2 & 11 \end{pmatrix}$
2. $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$
3. $\begin{pmatrix} -3 & 0 \\ 6 & 1 \end{pmatrix}$
4. $\begin{pmatrix} -5 & -4 \\ 1 & 1 \end{pmatrix}$
5. $\begin{pmatrix} 7 & 5 \\ -3 & -2 \end{pmatrix}$
6. $\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix}$
7. $\begin{pmatrix} 4 & -2 \\ 1 & 2 \end{pmatrix}$
8. $\begin{pmatrix} -2 & 6 \\ 0 & 5 \end{pmatrix}$
9. $\begin{pmatrix} 12 & 4 \\ 1 & 2 \end{pmatrix}$
10. $\begin{pmatrix} 7 & -8 \\ -3 & 2 \end{pmatrix}$

Check your answers with those at the end of the unit.

Solution of simultaneous equations using matrices

If we have two simultaneous equations, we may write these in matrix form as shown in this next example.

Example 9

Use matrices to solve the simultaneous equations below.

$$3x + 2y = 8$$
$$x + 4y = 6$$

We write these in matrix form as follows.

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

If we let,

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \qquad X = \begin{pmatrix} x \\ y \end{pmatrix} \qquad B = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

then,

$$AX = B,$$

and

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{12 - 2}\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 0.4 & -0.2 \\ -0.1 & 0.3 \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.4 & -0.2 \\ -0.1 & 0.3 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.4 \times 8 + (-0.2) \times 6 \\ (-0.1) \times 8 + 0.3 \times 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$x = 2$ and $y = 1$.

Try this exercise.

Exercise H

Use matrices to solve the following pairs of simultaneous equations.

1. $3x + y = 9$
   $2x + y = 7$
2. $-6x + 2y = 10$
   $2x + y = 0$
3. $2x + 3y = 9$
   $x + 2y = 5$
4. $-5x - 2y = -1$
   $-3x - 2y = 1$

Check your answers with those at the end of the unit.

Consider this last example.

Example 10

Given that $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1.5 \\ 0.5 & -0.5 \end{pmatrix}$

solve the equation,

$$AB\mathbf{p} = \mathbf{q}$$

if $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$.

We need to find AB.

$$AB = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1.5 \\ 0.5 & -0.5 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

so, $AB\mathbf{p} = \mathbf{q}$ is given by,

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$$

$$\mathbf{p} = (AB)^{-1}\mathbf{q}$$

$$(AB)^{-1} = \frac{1}{-2 - 3}\begin{pmatrix} -1 & -3 \\ -1 & 2 \end{pmatrix} = \frac{-1}{5}\begin{pmatrix} -1 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 0.2 & 0.6 \\ 0.2 & -0.4 \end{pmatrix}$$

$$\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.2 & 0.6 \\ 0.2 & -0.4 \end{pmatrix}\begin{pmatrix} 12 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \times 12 + 0.6 \times 1 \\ 0.2 \times 12 + (-0.4) \times 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$x = 3$ and $y = 2$.

Try this last exercise.

Exercise I

In this exercise $A = \begin{pmatrix} 0 & 5 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$

1. Solve the equation $AB\mathbf{p} = \mathbf{q}$, given that $\mathbf{q} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

2. Solve the equation $BA\mathbf{p} = \mathbf{c}$, given that $\mathbf{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

3. Solve the equation $AB^T\mathbf{p} = \mathbf{d}$, given that $\mathbf{d} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$.

Answers

Exercise A

1. $\begin{pmatrix} 5 & 8 & 9 \\ 7 & 3 & 6 \end{pmatrix}$ 2 x 3 matrix

2. $\begin{pmatrix} 1 & 3 \\ 2 & 2 \\ 2 & 0 \end{pmatrix}$ 3 x 2 matrix

3. $\begin{pmatrix} 0 & 13 & 8 & 12 & 2 \\ 18 & 0 & 7 & 8 & 3 \end{pmatrix}$ 2 x 5 matrix

4. $\begin{pmatrix} 185 & 177 & 202 & 206 \\ 215 & 191 & 176 & 184 \end{pmatrix}$ 2 x 4 matrix

5. $\begin{pmatrix} 6 & 2 & 0 & 0 & 2 & 0 \\ 2 & 3 & 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 0 & 1 & 2 \end{pmatrix}$ 3 x 6 matrix

Exercise B

1. $\begin{pmatrix} -2 & 21 \\ 32 & 0 \\ -5 & 6 \\ 7 & 8 \end{pmatrix}$

2. $\begin{pmatrix} 1 & 7 \\ 0 & 3 \\ -5 & 1 \end{pmatrix}$

3. $\begin{pmatrix} -2 & 14 & 10 \\ 36 & 0 & 9 \\ 15 & 3 & 12 \\ 7 & 24 & 11 \end{pmatrix}$

4. $\begin{pmatrix} 0 & 3 \\ -2 & 1 \end{pmatrix}$

5. $\begin{pmatrix} 12 & 21 \\ -5 & 0 \\ -2 & 6 \end{pmatrix}$

6. $\begin{pmatrix} -2 & 21 & 10 & 1 \\ 27 & 0 & 9 & -1 \end{pmatrix}$

Exercise C

1. (9 17 25)
2. (11 9 42 13)
3. (61 140)

4.a) $\begin{pmatrix} 4 & 3 & 6 \\ 6 & 7 & 3 \\ 2 & 2 & 3 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 4 & 3 & 6 \\ 6 & 7 & 3 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 14 & 13 & 15 \end{pmatrix}$

5.a) $\begin{pmatrix} 24 & 16 & 17 & 21 \\ 15 & 18 & 28 & 23 \end{pmatrix}$

b) $\begin{pmatrix} 1.25 & 1.20 \end{pmatrix}\begin{pmatrix} 24 & 16 & 17 & 21 \\ 15 & 18 & 28 & 23 \end{pmatrix}$

= (£48.00 £41.60 £54.85 £53.85)

Exercise D

1. $\begin{pmatrix} 6 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 15 \\ -4 & -3 \end{pmatrix}$

3. $\begin{pmatrix} 4 & 0 \\ 1 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 5 & 11 \\ -6 & -3 \\ -9 & 0 \end{pmatrix}$

5. Not possible

6. $\begin{pmatrix} 1 & 1 & 8 \\ 7 & 2 & 1 \end{pmatrix}$

7. Not possible

8. $\begin{pmatrix} 15 & 15 \\ 1 & -1 \\ 6 & 14 \end{pmatrix}$

9. $\begin{pmatrix} 25 & -18 & 7 \\ -11 & 7 & -2 \end{pmatrix}$

10. $\begin{pmatrix} -1 & -7 & 2 \\ 6 & -5 & 3 \\ -1 & -14 & 5 \end{pmatrix}$

Exercise E

1. $\begin{pmatrix} 6 & 2 \\ 4 & -2 \end{pmatrix}$

2. $\begin{pmatrix} -5 & -1 \\ 0 & 1 \end{pmatrix}$

3. $\begin{pmatrix} 1.5 & -0.5 \\ -11 & 0.5 \end{pmatrix}$

4. $\begin{pmatrix} -6 & -2 \\ -1 & 4 \end{pmatrix}$

5. $\begin{pmatrix} 1 & -4 \\ 2 & -1 \end{pmatrix}$

6. $\begin{pmatrix} -6 & 0 \\ 1 & -8 \end{pmatrix}$

7. $\begin{pmatrix} -14 & 1 \\ -2 & 14 \end{pmatrix}$

8. $\begin{pmatrix} -6 & -3 \\ -2 & 15 \end{pmatrix}$

9. $\begin{pmatrix} -4 & -6 \\ -2 & 21 \end{pmatrix}$

10. $\begin{pmatrix} 2 & -12 \\ -3 & 26 \end{pmatrix}$

Exercise F

1. -15
2. 27
3. -14
4. -10
5. 25

Exercise G

1. $\begin{pmatrix} 11 & -5 \\ -2 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 4/10 & 2/10 \\ 1/10 & 3/10 \end{pmatrix}$ or $\begin{pmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{pmatrix}$

3. $\begin{pmatrix} -1/3 & 0 \\ 2 & 1 \end{pmatrix}$

4. $\begin{pmatrix} -1 & -4 \\ 1 & 5 \end{pmatrix}$

5. $\begin{pmatrix} -2 & -5 \\ 3 & 7 \end{pmatrix}$

6. $\begin{pmatrix} 7/2 & -2 \\ -3/2 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3.5 & -2 \\ -1.5 & 1 \end{pmatrix}$

7. $\begin{pmatrix} 2/10 & 2/10 \\ -1/10 & 4/10 \end{pmatrix}$ or $\begin{pmatrix} 0.2 & 0.2 \\ -0.1 & 0.4 \end{pmatrix}$

8. $\begin{pmatrix} -5/10 & 6/10 \\ 0 & 2/10 \end{pmatrix}$ or $\begin{pmatrix} -0.5 & 0.6 \\ 0 & 0.2 \end{pmatrix}$

9. $\begin{pmatrix} 2/20 & -4/20 \\ -1/20 & 12/20 \end{pmatrix}$ or $\begin{pmatrix} 0.1 & -0.2 \\ -0.05 & 0.6 \end{pmatrix}$

10. $\begin{pmatrix} -2/10 & -8/10 \\ -3/10 & -7/10 \end{pmatrix}$ or $\begin{pmatrix} -0.2 & -0.8 \\ -0.3 & -0.7 \end{pmatrix}$

Exercise H

1. $x = 2$, $y = 3$
2. $x = -1$, $y = 2$
3. $x = 3$, $y = 1$
4. $x = 1$, $y = -2$

Exercise I

1. $x = 2$, $y = 1$
2. $x = -1$, $y = 2$
3. $x = 1$, $y = 1$