MATHS Xtra ALGEBRA

Unit 15

Partial fractions

Objectives

On completion of this unit you should understand:

1. Decomposition into four types of partial fractions.

2. Algebraic division.

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Partial fractions: Type 1

In the unit on algebraic fractions, we simplified,

$$\frac{2}{(x+1)} + \frac{3}{(x-1)}$$

by finding a common denominator of $(x + 1)(x - 1)$ and found that,

$$\frac{2}{(x+1)} + \frac{3}{(x-1)} = \frac{5x+1}{(x+1)(x-1)}.$$

We shall now cover the steps required to reverse this process.

Study these examples.

Example 1

Express $\frac{5x+1}{(x+1)(x-1)}$ in partial fractions.

If we multipied out the denominator, we would obtain, $x^2 - 1$. The highest power of x in the denominator is 2, and the highest power of x in the numerator is 1. (Note that $5x$ is $5x^1$). Providing the power of x in the numerator is **less than** that in the denominator, and the factors in the denominator are linear, we may proceed as follows.

We let,

$$\frac{5x+1}{(x+1)(x-1)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

where $\equiv$ means 'is equivalent to'.

We then take a common denominator, as follows.

$$\frac{5x+1}{(x+1)(x-1)} \equiv \frac{A(x-1) + B(x+1)}{(x+1)(x-1)}$$

The denominator on the left hand side, LHS, is then equal to the denominator on the right hand side, RHS. We can therefore equate the numerators.

$$5x + 1 \equiv A(x - 1) + B(x + 1)$$

We need to find the values of A and B.

To find A we let $x = -1$, so that $(x + 1) = 0$ and we have,

$$5(-1) + 1 = A(-1 - 1) + B(0)$$
$$-5 + 1 = -2A$$
$$-4 = -2A, \quad \text{so} \quad A = 2.$$

Similarly to find B, let $x = 1$, so that $(x - 1) = 0$ and we find that,

$$5(1) + 1 = A(0) + B(1 + 1)$$
$$6 = 2B, \quad \text{so} \quad B = 3.$$

We substitute our values back for A and B to find that,

$$\frac{5x+1}{(x+1)(x-1)} \equiv \frac{2}{(x+1)} + \frac{3}{(x-1)}.$$

Example 2

Express $\dfrac{x}{(x-4)(x-1)}$ in partial fractions.

If we multipied out the denominator, we would find that the highest power of x in the denominator is 2. The highest power of x in the numerator is 1. The power of x in the numerator is **less than** that in the denominator and the factors in the denominator are linear, so we may proceed as follows.

We let,

$$\frac{x}{(x-4)(x-1)} \equiv \frac{A}{(x-4)} + \frac{B}{(x-1)}$$

We take a common denominator, as follows.

$$\frac{x}{(x-4)(x-1)} \equiv \frac{A(x-1) + B(x-4)}{(x-4)(x-1)}$$

The denominator on the LHS is then equal to the denominator on the RHS. We can therefore equate the numerators.

$$x \equiv A(x-1) + B(x-4)$$

We need to find the values of A and B.

To find A we let $x = 4$, so that $(x - 4) = 0$ and we have,

$$4 = A(4-1) + B(0)$$
$$4 = 3A$$
$$A = \frac{4}{3}.$$

Similarly to find B, let $x = 1$, so that $(x - 1) = 0$ and we find that,

$$1 = A(0) + B(1-4)$$
$$1 = -3B$$
$$B = \frac{-1}{3}.$$

We substitute our values back for A and B to find that,

$$\frac{x}{(x-4)(x-1)} \equiv \frac{^4/_3}{(x-4)} + \frac{(^{-1}/_3)}{(x-1)}$$

or,

$$\frac{x}{(x-4)(x-1)} \equiv \frac{4}{3(x-4)} - \frac{1}{3(x-1)}.$$

Try the exercise on the next page.

Exercise A

Express each of the following as partial fractions.

1. $\dfrac{4x}{(x+1)(x-3)}$

2. $\dfrac{6}{(x-6)(x-2)}$

3. $\dfrac{3x}{(x-4)(2x-3)}$

4. $\dfrac{2x}{(x+6)(x-2)}$

5. $\dfrac{2}{(2x+3)(x-5)}$

6. $\dfrac{3x+1}{(x+2)(x-5)}$

7. $\dfrac{4x-3}{(x+1)(x-3)}$

8. $\dfrac{-8}{(x+1)(x-3)}$

9. $\dfrac{x-1}{(x+1)(x-3)}$

10. $\dfrac{6x}{(x+6)(x-6)}$

Check your answers with those at the end of the unit.
Study these examples.

Example 3

Decompose $\dfrac{2}{5x(x+4)(x-1)}$ into partial fractions.

The power of x in the numerator is less than the power of x in the denominator, the factors are linear, so we proceed as follows.

$$\frac{2}{5x(x+4)(x-1)} \equiv \frac{A}{5x} + \frac{B}{(x+4)} + \frac{C}{(x-1)}$$

We take a common denominator, as follows.

$$\frac{2}{5x(x+4)(x-1)} \equiv \frac{A(x+4)(x-1) + B(5x)(x-1) + C(5x)(x+4)}{5x(x+4)(x-1)}$$

The denominator on the LHS is equal to the denominator on the RHS.
Equate the numerators.

$$2 \equiv A(x+4)(x-1) + B(5x)(x-1) + C(5x)(x+4)$$

Let $x = -4$
$2 = 0 + B(5)(-4)(-4-1) + 0$
$2 = B(5)(-4)(-5)$
$2 = 100B$
$B = {}^{1}/_{50}$

Let $x = 1$
$2 = 0 + 0 + C(5)(1)(1+4)$
$2 = C(5)(1)(5)$
$2 = 25C$
$C = {}^{2}/_{25}$

Let $x = 0$
$2 = A(0+4)(0-1) + 0 + 0$
$2 = A(4)(-1)$
$2 = -4A$
$A = {}^{-1}/_{2}$

$$\frac{2}{5x(x+4)(x-1)} \equiv \frac{-1}{10x} + \frac{1}{50(x+4)} + \frac{2}{25(x-1)}.$$

Example 4

Express $\dfrac{x+2}{(x+1)(x^2-9)}$ in partial fractions.

This example appears to contain a quadratic factor, $(x^2 - 9)$ in the denominator. In fact $(x^2 - 9)$ will factorise into two linear factors, $(x - 3)(x + 3)$.

We proceed as follows,

$$\frac{x+2}{(x+1)(x^2-9)} = \frac{x+2}{(x+1)(x-3)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)} + \frac{C}{(x+3)}$$

We can continue to find A, B and C as before.

$$\frac{x+2}{(x+1)(x-3)(x+3)} = \frac{A(x-3)(x+3) + B(x+1)(x+3) + C(x+1)(x-3)}{(x+1)(x-3)(x+3)}$$

Equate the numerators.

$$x+2 = A(x-3)(x+3) + B(x+1)(x+3) + C(x+1)(x-3)$$

Let $x = 3$	Let $x = -3$	Let $x = -1$
$5 = B(4)(6)$	$-1 = C(-2)(-6)$	$1 = A(-4)(2)$
$5 = 24B$	$-1 = 12C$	$1 = -8A$
$B = \dfrac{5}{24}$	$C = \dfrac{-1}{12}$	$A = \dfrac{-1}{8}$

$$\frac{x+2}{(x+1)(x^2-9)} = \frac{-1}{8(x+1)} + \frac{5}{24(x-3)} - \frac{1}{12(x+3)}.$$

Try this exercise.

Exercise B

Decompose the following into partial fractions.

1. $\dfrac{1}{x(x-2)(x+1)}$
2. $\dfrac{3}{2x(x-2)(x+5)}$
3. $\dfrac{4x+1}{x(x+4)(2x+1)}$
4. $\dfrac{x-1}{3x(x+2)(2x+1)}$
5. $\dfrac{3x}{(x-1)(x-3)(x+1)}$
6. $\dfrac{2x+1}{(x-3)(x^2-4)}$
7. $\dfrac{x-1}{(x^2-16)(x+1)}$
8. $\dfrac{6x}{(x^2-1)(x+3)}$

Check your answers with those at the end of the unit.

Partial fractions: Type 2

Consider this example.

Example 5

Express $\dfrac{x+3}{(x+1)(x^2+2)}$ in partial fractions.

This example contains a quadratic factor, $(x^2 + 2)$ in the denominator. It will not factorise into linear factors. The highest power of x in the numerator is less than that in the denominator, so we proceed in the following way to allow for the power of x in the numerator to be one less than the power of x in the denominator.

Let,

$$\frac{x+3}{(x+1)(x^2+2)} \equiv \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+2)}$$

Take a common denominator as before.

$$\frac{x+3}{(x+1)(x^2+2)} \equiv \frac{A(x^2+2) + (Bx+C)(x+1)}{(x+1)(x^2+2)}$$

Equate the numerators.

$$x+3 \equiv A(x^2+2) + (Bx+C)(x+1)$$

Let $x = -1$

$-1 + 3 = A(1 + 2)$

$2 = 3A$ and therefore $A = \dfrac{2}{3}$.

We now need to find B and C. We cannot find a value for x to eliminate the bracket $(x^2 + 2)$. We need to use another method. We shall compare coefficients.

$$x+3 \equiv A(x^2+2) + (Bx+C)(x+1)$$

Start with coefficients of x^2. We can see that there is no x^2 term on the LHS, but on the RHS, the coefficients of x^2 are A and B. We can see this without multiplying the brackets out. So,

$0 = A + B$

$A = \dfrac{2}{3}$, so $B = \dfrac{-2}{3}$.

Compare the number terms. The number on the LHS is 3. 2A and C are the numbers on the RHS. (This is the same as letting $x = 0$.)

$3 = 2A + C$

$3 = 2 \times \dfrac{2}{3} + C$ and therefore $C = 3 - \dfrac{4}{3} = \dfrac{9}{3} - \dfrac{4}{3} = \dfrac{5}{3}$.

We substitute these values back as follows.

$$\frac{x+3}{(x+1)(x^2+2)} \equiv \frac{2}{3(x+1)} + \frac{(-2x+5)}{3(x^2+2)}.$$

Try this exercise.

Exercise C

Decompose the following into partial fractions.

1. $\dfrac{1}{(x-2)(x^2+1)}$
2. $\dfrac{3}{(x-2)(x^2+5)}$
3. $\dfrac{x-1}{(x^2+4)(2x+1)}$
4. $\dfrac{x-2}{(x^2+2)(2x+1)}$
5. $\dfrac{3x}{(x-1)(x^2-3)}$
6. $\dfrac{2x+1}{(x-3)(x^2+4)}$
7. $\dfrac{x-1}{(x^2+3)(x+1)}$
8. $\dfrac{6x}{(x^2+1)(x+3)}$

Check your answers with those at the end of the unit.

Partial fractions: Type 3

Consider these examples.

Example 6

Express $\dfrac{x+3}{x^2(x+2)}$ in partial fractions.

x^2 is a repeated factor. The substitution we use is as follows.

$$\frac{x+3}{x^2(x+2)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+2)}$$

Take a common denominator. This is $x^2(x+2)$ **not** $(x)(x^2)(x+2)$.

$$\frac{x+3}{x^2(x+2)} \equiv \frac{Ax(x+2) + B(x+2) + Cx^2}{x^2(x+2)}$$

Equate the numerators.

$$x+3 \equiv Ax(x+2) + B(x+2) + Cx^2$$

Let $x = -2$,

$1 = 4C$ $\qquad C = \frac{1}{4}.$

Coefficients of x^2,

$0 = A + C$ $\qquad A = \frac{-1}{4}.$

Numbers, or $x = 0$,

$3 = 2B$ $\qquad B = \frac{3}{2}.$

so,

$$\frac{x+3}{x^2(x+2)} \equiv \frac{-1}{4x} + \frac{3}{2x^2} + \frac{1}{4(x+2)}.$$

Example 7

Write the form of partial fraction required for each of the following.

a) $\dfrac{2x + 5}{(x + 2)^2(2x + 1)}$

b) $\dfrac{6x}{(x + 1)^2(2x + 1)}$

c) $\dfrac{3x - 4}{(x + 2)(x^2)}$

a) The repeated factor is $(x + 2)^2$ so the partial fraction is as follows.

$$\frac{2x + 5}{(x + 2)^2(2x + 1)} \equiv \frac{A}{(x + 2)} + \frac{B}{(x + 2)^2} + \frac{C}{(2x + 1)}$$

b) The repeated factor is $(x + 1)^2$ so the partial fraction is as follows.

$$\frac{6x}{(x + 1)^2(2x + 1)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 1)^2} + \frac{C}{(2x + 1)}$$

c) The repeated factor is x^2 so the partial fraction is as follows.

$$\frac{3x - 4}{(x + 2)(x^2)} \equiv \frac{A}{(x + 2)} + \frac{B}{x} + \frac{C}{x^2}$$

Try this exercise.

Exercise D

Write the partial fraction required for each of the following.

1. $\dfrac{2x + 7}{(x + 3)^2(x - 1)}$
2. $\dfrac{6x}{x^2(2x + 4)}$
3. $\dfrac{x - 4}{(x + 2)(x - 1)^2}$

Express the following as partial fractions.

4. $\dfrac{x + 5}{(x - 2)^2(2x + 1)}$
5. $\dfrac{x}{(x - 1)^2(3x + 1)}$
6. $\dfrac{3x - 4}{(x - 2)(x^2)}$

Check your answers with those at the end of the unit.

Partial fractions: Type 4

Consider this example.

Example 8

Express $\dfrac{2x^2 + 3}{(x + 2)(x - 1)}$ in partial fractions.

If we multiply out the denominator we get $x^2 + x - 2$. You should notice that the highest power of x in the numerator is the **same** as that in the denominator. The highest power in the numerator must be **less than** that of the denominator for the expression to be a fraction.

Before continuing we must divide x^2 into $2x^2$ to obtain 2, as follows.

$$\frac{2x^2 + 3}{(x + 2)(x - 1)} \equiv 2 + \frac{A}{(x + 2)} + \frac{B}{(x - 1)}$$

Take a common denominator as before.

$$\frac{2x^2 + 3}{(x + 2)(x - 1)} \equiv \frac{2(x + 2)(x - 1) + A(x - 1) + B(x + 2)}{(x + 2)(x - 1)}$$

Equate the numerators.

$$2x^2 + 3 \equiv 2(x + 2)(x - 1) + A(x - 1) + B(x + 2)$$

Let $x = -2$	Let $x = 1$
$2(-2)^2 + 3 = A(-2 - 1)$	$2(1)^2 + 3 = B(1 + 2)$
$11 = -3A$	$5 = 3B$
$A = \dfrac{-11}{3}$	$B = \dfrac{5}{3}$

Substitute these values back.

$$\frac{2x^2 + 3}{(x + 2)(x - 1)} \equiv 2 - \frac{11}{3(x + 2)} + \frac{5}{3(x - 1)}.$$

Try this short exercise.

Exercise E

1. Express $\dfrac{2x^2 + 5}{(x + 4)(x - 2)}$ in partial fractions.
2. Express $\dfrac{3x^2 - 4}{(x + 1)(x - 1)}$ in partial fractions.
3. Express $\dfrac{4x^2 + 1}{(x + 3)(x - 2)}$ in partial fractions.

Check your answers with those at the end of the unit.

Consider these examples, which illustrate algebraic division.

Example 9

Express $\dfrac{4x^3 + 3x^2 - 4x + 1}{(x + 2)(x - 3)}$ in partial fractions.

If we multiply out the denominator we get $x^2 - x - 6$. You should notice that the highest power of x in the numerator is the **higher than** that in the denominator. The highest power in the numerator must be **less than** that of the denominator for the expression to be a fraction.

The method we used in Example 8, when the power of the numerator was the **same** as that of the denominator, will **not** apply, we must use algebraic division as follows.

$$x^2 - x - 6 \,\overline{)\,4x^3 + 3x^2 - 4x + 1}$$

We need to eliminate the highest term first. We ask, what we need to multiply x^2 by to obtain $4x^3$?
Answer $4x$.

$$\begin{array}{r} 4x \\ x^2 - x - 6 \,\overline{)\,4x^3 + 3x^2 - 4x + 1} \\ \underline{4x^3 - 4x^2 - 24x} \\ 7x^2 + 20x + 1 \end{array}$$

Write $4x$ as shown and multiply $x^2 - x - 6$ by $4x$ to obtain $4x^3 - 4x^2 - 24x$.
Take this answer from $4x^3 + 3x^2 - 4x + 1$.
Note that
$3x^2 - (-4x^2) = 7x^2$ and $-4x - (-24x) = 20x$.

We now need to eliminate the next highest term. We ask, what we need to multiply x^2 by to obtain $7x^2$?
Answer $+7$.

$$\begin{array}{r} 4x + 7 \\ x^2 - x - 6 \,\overline{)\,4x^3 + 3x^2 - 4x + 1} \\ \underline{4x^3 - 4x^2 - 24x} \\ 7x^2 + 20x + 1 \\ \underline{7x^2 - 7x - 42} \\ 27x + 43 \end{array}$$

Write $+7$ as shown and multiply $x^2 - x - 6$ by 7 to obtain $7x^2 - 7x - 42$.
Take this answer from $7x^2 + 20x + 1$. We are now left with a remainder of $27x + 43$.
The highest power of the remainder is less than that of $x^2 - x - 6$ so our division is complete.

We can now write that,

$$\frac{4x^3 + 3x^2 - 4x + 1}{(x + 2)(x - 3)} \equiv 4x + 7 + \frac{27x + 43}{(x + 2)(x - 3)}$$

Decompose this into partial fractions as before using Type 1.

$$\frac{4x^3 + 3x^2 - 4x + 1}{(x + 2)(x - 3)} \equiv 4x + 7 + \frac{11}{5(x + 2)} + \frac{124}{5(x - 3)}$$

Example 10

Express $\dfrac{-2x^3 + 3x^2 - 3x + 5}{(x+1)(x-1)}$ in partial fractions.

If we multiply out the denominator we get $x^2 - 1$. The highest power of x in the numerator is the **higher than** that in the denominator. We must use algebraic division.

$$x^2 - 1 \overline{\smash{\big)} -2x^3 + 3x^2 - 3x + 5}$$

We need to eliminate the highest term first. We ask, what we need to multiply x^2 by to obtain $-2x^3$? Answer $-2x$.

$$\begin{array}{r l}
 & \quad -2x \\
x^2 - 1 & \overline{\big) -2x^3 + 3x^2 - 3x + 5} \\
 & \underline{\;\; -2x^3 \qquad\quad + 2x \quad} \\
 & \qquad\quad 3x^2 - 5x + 5
\end{array}$$

Write $-2x$, as shown and multiply $x^2 - 1$ by $-2x$ to obtain $-2x^3 + 2x$. Take this answer from $-2x^3 + 3x^2 - 3x + 5$. Line up $-2x^3$ with $-2x^3$ and $+2x$ with $-3x$.

We now need to eliminate the next highest term. We ask, what we need to multiply x^2 by to obtain $3x^2$? Answer $+3$.

$$\begin{array}{r l}
 & \quad -2x + 3 \\
x^2 - 1 & \overline{\big) -2x^3 + 3x^2 - 3x + 5} \\
 & \underline{\;\; -2x^3 \qquad\quad - 2x \quad} \\
 & \qquad\quad 3x^2 - 5x + 5 \\
 & \qquad\quad \underline{3x^2 \qquad\; - 3} \\
 & \qquad\qquad\quad - 5x + 8
\end{array}$$

Write $+3$, as shown and multiply $x^2 - 1$ by 3 to obtain $3x^2 - 3$. Take this answer from $3x^2 - x + 5$. We are now left with a remainder of $-5x + 8$. The highest power of the remainder is less than that of $x^2 - 1$ so our division is complete.

We can now write that,

$$\frac{-2x^3 + 3x^2 - 3x + 5}{(x+1)(x-1)} \equiv -2x + 3 + \frac{(-5x + 8)}{(x+1)(x-1)}$$

Decompose this into partial fractions as before.

$$\frac{-2x^3 + 3x^2 - 3x + 5}{(x+1)(x-1)} \equiv -2x + 3 - \frac{13}{2(x+1)} + \frac{3}{2(x-1)}.$$

Try the exercise on the next page.

Exercise F

1. Express $\dfrac{-x^3 + 4x^2 - 2x + 3}{(x + 1)(x - 2)}$ in partial fractions.

2. Express $\dfrac{-2x^3 + 5x^2 + x + 5}{(x + 3)(x - 1)}$ in partial fractions.

3. Express $\dfrac{4x^3 + 5x^2 - 3x + 5}{(x - 1)(x - 4)}$ in partial fractions.

4. Express $\dfrac{5x^3 - 3x^2 - 3x + 2}{(x + 1)(x - 1)}$ in partial fractions.

Check your answers with those at the end of the unit.

Answers

Exercise A

1. $\frac{1}{(x+1)} + \frac{3}{(x-3)}$
2. $\frac{3}{2(x-6)} - \frac{3}{2(x-2)}$
3. $\frac{12}{5(x-4)} - \frac{9}{5(2x-3)}$
4. $\frac{3}{2(x+6)} + \frac{1}{2(x-2)}$
5. $\frac{-4}{13(2x+3)} + \frac{2}{13(x-5)}$
6. $\frac{5}{7(x+2)} + \frac{16}{7(x-5)}$
7. $\frac{7}{4(x+1)} + \frac{9}{4(x-3)}$
8. $\frac{-2}{(x+1)} + \frac{2}{(x-3)}$
9. $\frac{1}{2(x+1)} + \frac{1}{2(x-3)}$
10. $\frac{3}{(x+6)} + \frac{3}{(x+6)}$

Exercise B

1. $\frac{-1}{2x} + \frac{1}{6(x-2)} + \frac{1}{3(x+1)}$
2. $\frac{-3}{20x} + \frac{3}{28(x-2)} + \frac{3}{70(x+5)}$
3. $\frac{1}{4x} - \frac{15}{28(x+4)} + \frac{4}{7(2x+1)}$
4. $\frac{-1}{6x} - \frac{1}{6(x+2)} + \frac{2}{3(2x+1)}$
5. $\frac{-3}{4(x-1)} + \frac{9}{8(x-3)} - \frac{3}{8(x+1)}$
6. $\frac{7}{5(x-3)} - \frac{3}{20(x+2)} - \frac{5}{4(x-2)}$
7. $\frac{3}{40(x-4)} - \frac{5}{24(x+4)} + \frac{2}{15(x+1)}$
8. $\frac{3}{4(x-1)} + \frac{3}{2(x+1)} - \frac{9}{4(x+3)}$

Exercise C

1. $\frac{1}{5(x-2)} + \frac{(-x-2)}{5(x^2+1)}$
2. $\frac{1}{3(x-2)} + \frac{(-x-2)}{3(x^2+5)}$
3. $\frac{3x+7}{17(x^2+4)} - \frac{6}{17(2x+1)}$
4. $\frac{5x+2}{9(x^2+2)} - \frac{10}{9(2x+1)}$
5. $\frac{-3}{2(x-1)} + \frac{3x+9}{2(x^2-3)}$
6. $\frac{7}{13(x-3)} + \frac{(-7x+5)}{13(x^2+4)}$
7. $\frac{x+1}{2(x^2+3)} - \frac{1}{2(x+1)}$
8. $\frac{9x+3}{5(x^2+1)} - \frac{9}{5(x+3)}$

Exercise D

1. $\frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{C}{(x-1)}$
2. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+4)}$
3. $\frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$
4. $\frac{-9}{25(x-2)} + \frac{7}{5(x-2)^2} + \frac{18}{25(2x+1)}$
5. $\frac{1}{16(x-1)} + \frac{1}{4(x-1)^2} - \frac{3}{16(3x+1)}$
6. $\frac{1}{2(x-2)} - \frac{1}{2x} + \frac{2}{x^2}$

Exercise E

1. $2 - \frac{37}{6(x+4)} + \frac{13}{6(x-2)}$
2. $3 + \frac{1}{2(x+1)} - \frac{1}{2(x-1)}$
3. $4 - \frac{37}{5(x+3)} + \frac{17}{5(x-2)}$

Exercise F

1. $-x+3+\frac{-x+9}{(x+1)(x-2)} \equiv -x+3-\frac{10}{3(x+1)}+\frac{7}{3(x-2)}$
2. $-2x+9+\frac{-23x+32}{(x+3)(x-1)} \equiv -2x+9-\frac{101}{4(x+3)}+\frac{9}{4(x-1)}$
3. $4x+25+\frac{106x-95}{(x-1)(x-4)} \equiv 4x+25-\frac{11}{3(x-1)}+\frac{329}{3(x-4)}$
4. $5x-3+\frac{2x-1}{(x+1)(x-1)} \equiv 5x-3+\frac{3}{2(x+1)}+\frac{1}{2(x-1)}$