MATHS Xtra
ALGEBRA

Unit 1

Basic algebra

Objectives
On completion of this unit you should be able to:

1. Collect like terms.

2. Understand directed numbers.

3. Solve simple linear equations.

4. Solve simple linear equations which contain brackets.

Collection of like terms

Consider what happens if you are told to add,

$$7 + 7 + 7 + 7 + 7$$

We can add these up, or alternatively we can say that, we have the number 7 five times, and calculate,

$$5 \times 7$$

If our number 7 had been an unknown number, x, and we are told to collect like terms together,

$$x + x + x + x + x$$

We now have five times the number x. This is,

$$5 \times x$$

and is written as $5x$ for simplicity.

Consider this example.

Example 1

Collect the following terms together.

    a) $y + y + 2y + 3y + 5y$
    b) $x + 3x + 2y + 4y$
    c) $23 + 5x - 10 - 2x$

a) $y + y + 2y + 3y + 5y$
Count the number of y's.

$$1 + 1 + 2 + 3 + 5 = 12 \quad \text{So,}$$
$$y + y + 2y + 3y + 5y = 12y.$$

b) $x + 3x + 2y + 4y$
First consider the x terms.

$$x + 3x$$

This is equal to $4x$.
Then consider the y terms.

$$2y + 4y$$

This is equal to $6y$. So,

$$x + 3x + 2y + 4y = 4x + 6y. \qquad \text{Note that this can also be written as,}$$
$$6y + 4x.$$

c) $23 + 5x - 10 - 2x$
Count the numbers first,

$$23 - 10 = 13$$

Then the x terms,

$$5x - 2x = 3x \quad \text{So,}$$
$$23 + 5x - 10 - 2x = 13 + 3x.$$
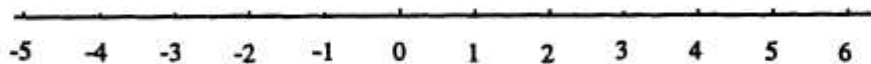
Try this exercise.

Exercise A
In each case, collect like terms.
1. $2x + 3x - 5x - x + 12x$
2. $3y + 4y + 8y - 2y$
3. $5x + 2y + 8y - 4y + 7x$
4. $7x + 4y + 9y - 8x$
5. $2x + 5y + 2y - 8x$
6. $25 + 8y + 2y + 5$
7. $42x + y + 10 + 3y - 39x$
8. $12 + 6 + 5x + 10x$
9. $15 + 5y + 6y + 7$
10. $21 + 2y + 4x - 4y$

Check your answers with those at the end of the unit.

Directed numbers

To understand directed numbers we need a number line.

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-5   -4   -3   -2   -1   0   1   2   3   4   5   6
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Example 2
Using the number line, calculate the following,

    a) $2 + 3$
    b) $-2 + 6$
    c) $-4 + 2$
    d) $-2 - 3$
    e) $-3a - a$

a)   Start at the number 2 and add 3 by moving three places to the right. The answer is 5.

b)   Start at the number -2. Add 6 by moving 6 places to the right. The answer is 4.

c)   Start at the number -4. Add 2 by moving 2 places to the right. The answer is -2.

d)   Start at the number -2. Subtract 3 by moving 3 places to the left. The answer is -5.

e)   The same rules apply. Start at -3, subtract 1 by moving one place to the left. We obtain the number -4. The answer is $-4a$.

Try this short exercise using the number line. You may need to add extra numbers to it.

Exercise B

1.	$-2 + 4$	6.	$-2y + 3y - y$
2.	$-3 - 4$	7.	$-x + x - 2x$
3.	$-a + 3a$	8.	$3y + y$
4.	$y - 4y$	9.	$2 - 4 - 1$
5.	$-2 + 3 - 5$	10.	$3x - 5x$

Check your answers with those at the end of the unit.

Making simple equations

If we know the value of an algebraic expression we have an equation.

Study these examples.

Example 3
Write an equation for,
'the number that is 8 more than x is 12.'

The number that is 8 more than x, is written as the expression $x + 8$.
We can now make the equation.
The number that is 8 more than x equals 12.
$$x + 8 = 12$$
If you think about this you will see that x is equal to 4.

Example 4
Write an equation for,
'the number that is 4 times x is 20.

The number that is '4 times x' is written $4x$.
We can now write the equation.
The number that is 4 times x equals 20.
$$4x = 20$$
Think about this and you will see that x is equal to 5.

Example 5
Bert buys 10 gallons of petrol and 6 litres of oil at a garage. The total cost is £35. The cost of a gallon of petrol is represented by £m and the cost of a litre of oil is represented by £q, write an equation to represent Bert's purchases.

10 gallons of petrol cost £$10m$.
6 litres of oil cost £$6q$.
The total cost is £35, so we can say that,
$$10m + 6q = 35.$$

Try this exercise.

Exercise C
Write equations for each of the following.

1. The number that is 4 more than x is 7.

2. The number that is twice x is 5.

3. The number that is half y is 8.

4. The number that is 6 less than y is 4.

5. The number that is a quarter of x is 2.

6. The cost of a gas bill is made up of a standing charge of £15 plus 10 pence per unit. If the total cost is represented by £C and the number of units is represented by U, write an equation for the total cost C.

7. Susan buys 7 apples at x pence each and 4 bananas at y pence each. If the total bill is 100 pence, write an equation for this.

8. The cost of a train ticket for an adult is £x. A child's ticket costs £y. If Mike takes his two children on the train and the total cost is £30, write an equation for this in terms of x and y.

9. Jarved pays £x for his bus ticket. The following week, when he buys a ticket, the price has risen by £2. Write an equation for the total cost, £T, of the two tickets.

10. Fred runs x miles. John runs three times as far. If the distance John runs is represented by y, write an equation for the distance John runs in terms of x and y.

Check your answers with those at the end of the unit.

Rules
When solving equations, there are two rules which need to be observed.
1. Both sides of an equation must be balanced.
2. You must keep the equation balanced by doing the same thing to both sides.

Consider these examples.

Example 6
Find x if,

$$x + 9 = 13$$

We need to take 9 from the left hand side to leave x, so, to keep the equation balanced, we must take 9 from the right hand side as well.

$$x + 9 - 9 = 13 - 9$$
$$x = 4.$$

Example 7
Find x if,

$$x + 5 = 20$$

To leave x on its own we need to take 5 away from both sides of the equation.

$$x + 5 - 5 = 20 - 5$$
$$x = 15.$$

Example 8
Solve this equation.

$$y - 9 = 15$$

We are told to solve the equation. This means we need to find the value of y. To have y on its own we need to add 9 to both sides.

$$y - 9 + 9 = 15 + 9$$
$$y = 24.$$

Example 9
Solve this equation.

$$9x = 36$$

We need to find the value of x this time.
Remember that $9x$ means 9 lots of x. The x is multiplied by 9.
We need to divide both sides by 9.

$$\frac{9x}{9} = \frac{36}{9}$$

$$x = 4.$$

Example 10
Find q if,

$$\frac{q}{8} = 20$$

This time q is divided by 8.
We need to multiply both sides by 8.

$$\frac{q}{8} \times 8 = 20 \times 8$$

$$q = 160.$$

Try this exercise.

Exercise D
Solve these equations.
1. $x + 5 = 25$
2. $y - 11 = 21$
3. $x + 6 = 4$
4. $2y = 20$
5. $6x = 42$
6. $\frac{x}{5} = 3$
7. $\frac{p}{4} = 21$
8. $7q = 63$
9. $5t = 35$
10. $\frac{x}{5} = -4$

Check your answers with those at the end of the unit.

Now consider the examples on the next page.

Example 11
Find x if,
$$3x - 2 = 13$$

We need the term involving x on its own, so we start by adding 2 to both sides.
$$3x - 2 + 2 = 13 + 2$$
$$3x = 15$$
We want to find x, and x is multiplied by 3, so we now divide both sides by 3.
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5.$$

Example 12
Find y if,
$$6y + 8 = 26$$

First obtain the term containing y on its own. To do this we need to take 8 from each side.
$$6y + 8 - 8 = 26 - 8$$
$$6y = 18$$
y is multiplied by 6, so we divide both sides by 6.
$$\frac{6y}{6} = \frac{18}{6}$$
$$y = 3.$$

Now try this exercise yourself.

Exercise E
Solve the following equations.

1. $3y + 5 = 14$
2. $2x - 6 = 8$
3. $5y + 7 = 27$
4. $4y - 4 = 24$
5. $3y + 2 = 20$

6. $2x + 8 = 5$
7. $3y - 2 = 7$
8. $12x + 4 = 52$
9. $11x + 6 = 94$
10. $4x - 10 = 2$

Check your answers with those at the end of the booklet.

Multiplying out brackets

Example 13
Multiply out the bracket in the following.
$4(x + 3)$

The number 4 outside the bracket means that everything inside the bracket is multiplied by 4. We can multiply out the bracket as follows,
4 x x + 4 x 3
So we have,
$4x + 12$.

Example 14
Multiply out the bracket in the following.
$3(2y - 6)$

Multiply everything inside the bracket by the number 3.
3 x $2y$ - 3 x 6
We have,
$6y - 18$.

Example 15
Multiply out the bracket in the following.
$-5(4y - 6)$

This time we need to multiply everything inside the bracket by the number -5.

$$-5 \text{ x } 4y - 5 \text{ x } -6$$

We need another little rule here.

-	x	+	is	-
+	x	-	is	-
-	x	-	is	+

So,
$$-5 \text{ x } 4y = -20y,$$
and
$$-5 \text{ x } -6 = +30$$

$$-5 \text{ x } 4y - 5 \text{ x } -6 = -20y + 30.$$

Try the exercise on the next page.

Exercise F

Multiply out the brackets and collect like terms where appropriate.

1. $2(3x + 4)$
2. $3(2x + 5)$
3. $-3(3x + 4)$
4. $-4(x - 3)$
5. $5(2y - 7)$
6. $3(x + 2) + 5(3x + 8)$
7. $3(2x + 8) - 3(3x + 4)$
8. $2(5y + 5) - 2(4x - 8)$
9. $3(3y - 7) + 5(2y + 4)$
10. $3(x - 6) - 5(x - 3)$

Check your answers with those at the end of the unit.

Solving simple equations

Again we need a set of rules to follow.

Rules

1. Remove any brackets.
2. Simplify each side of the equation if it is possible.
3. Solve the equations, as before, keeping each side balanced.

Study these examples.

Example 16

Find x if,

$6(x + 3) - 2 = 52$

Multiply out the bracket.

$6x + 18 - 2 = 52$

Simplify the left hand side of the equation by collecting like terms.

$6x + 16 = 52$

We now need to take 16 from each side of the equation to obtain the x term on its own.

$6x + 16 - 16 = 52 - 16$

$6x = 36$

The x is multiplied by 6. Divide both sides of the equation by 6.

$$\frac{6x}{6} = \frac{36}{6}$$

$x = 6.$

Example 17
Find x, if,
$2(x + 5) - 3(2x - 20) = -10$

$2(x + 5) - 3(2x - 20) = -10$
Multiply out the brackets.
$2x + 10 - 6x + 60 = -10$
Collect like terms together.
$-4x + 70 = -10$
Take 70 from both sides to leave the term containing x on its own.
$-4x + 70 - 70 = -10 - 70$
$-4x = -80$
Multiply both sides by -1.
$4x = 80$
Divide both sides by 4.
$$\frac{4x}{4} = \frac{80}{4}$$
$x = 20.$

Try this exercise.

Exercise G
Solve each of the following equations.
1. $2(x + 3) = 12$
2. $3(x - 7) = 15$
3. $4(2x - 1) = 20$
4. $5(3x - 7) = 25$
5. $2(4x - 3) = 34$
6. $2(5x + 11) = 52$
7. $3(x + 20) = 96$
8. $5(x + 7) = 45$
9. $7(x + 2) = 49˙$
10. $9(x + 6) = 36$

Check your answers with those at the end of the unit.

Answers

Exercise A
1. $11x$
2. $13y$
3. $12x + 6y$
4. $-x + 13y$ or $13y - x$
5. $-6x + 7y$ or $7y - 6x$
6. $30 + 10y$
7. $3x + 4y + 10$
8. $18 + 15x$
9. $22 + 11y$
10. $21 - 2y + 4x$

Exercise B
1. 2
2. -7
3. $2a$
4. $-3y$
5. -4
6. 0
7. $-2x$
8. $4y$
9. -3
10. $-2x$

Exercise C
1. $x + 4 = 7$
2. $2x = 5$
3. $\dfrac{y}{2} = 8$
4. $y - 6 = 4$
5. $\dfrac{x}{4} = 2$
6. $C = 15 + \dfrac{10U}{100}$
   or $C = 15 + 0.1U$
7. $7x + 4y = 100$
8. $x + 2y = 30$
9. $T = x + (x + 2)$
   or $T = 2x + 2$
10. $y = 3x$

Exercise D
1. $x = 20$
2. $y = 32$
3. $x = -2$
4. $y = 10$
5. $x = 7$
6. $x = 15$
7. $p = 84$
8. $q = 9$
9. $t = 7$
10. $x = -20$

Exercise E
1. $y = 3$
2. $x = 7$
3. $y = 4$
4. $y = 7$
5. $y = 6$
6. $x = -1.5$
7. $y = 3$
8. $x = 4$
9. $x = 8$
10. $x = 3$

Exercise F
1. $6x + 8$
2. $6x + 15$
3. $-9x - 12$
4. $-4x + 12$
5. $10y - 35$
6. $18x + 46$
7. $-3x + 12$
8. $10y - 8x + 26$
9. $19y - 1$
10. $-2x - 3$

Exercise G
1. $x = 3$
2. $x = 12$
3. $x = 3$
4. $x = 4$
5. $x = 5$
6. $x = 3$
7. $x = 12$
8. $x = 2$
9. $x = 5$
10. $x = -2$