Graphical Representation of Linear Inequalities

Inequalities

Signs such as <, ≤, >, ≥ are called inequality signs.

Sign	Meaning
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

If we are told that $x \geq 0$, what does this tell us about the numerical value of x? There are many values of x which will satisfy this inequality, *e.g.* $x=1$, $x=2$, $x=3$, … In fact there are so many values which satisfy this inequality that it is impossible to list them all, but we can show them graphically.

Graphically Representing Linear Inequalities

All the following points satisfy the inequality $x \geq 0$: (1,1), (3,4), (5,2), (2,-2), (4,-4)

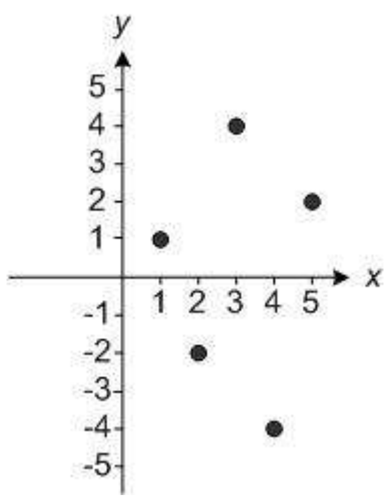

Fig. 1

We could find any number of other points which would also satisfy the inequality. If we attempted to represent all these points we would end up with a shaded region.

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Written by Gareth Lotwick, March 2006

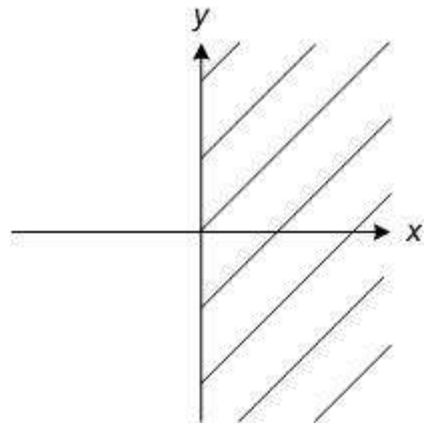


Fig. 2

The boundary of the region would be along the vertical line at x=0, *i.e*. along the y axis. Since $x \geq 0$ includes all points where x is zero or more, the region includes all the points on the boundary. The region goes on forever in all other directions.

Example 1

On separate diagrams shade the regions

a) $y < 0$
b) $x + y > 3$

Then show the region $x \geq 0$, $y < 0$, $x + y > 3$ on a single diagram.

Solution

a) $y < 0$ includes all points where y is any value up to but **not** including zero. So the region is the shaded area shown. This does **not** include points on the axis of x, which we usually show by drawing this 'boundary' as a dotted line.

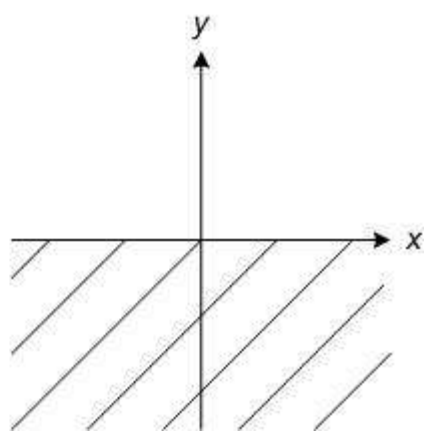


Fig. 3

b) *How do we find the region corresponding to* $x + y > 3$?

Well, $x + y = 3$ is a straight line, and will form the boundary of our region. We can draw this as follows:

Putting $x = 0$, then $y = 3$.
This means that (0,3) is a point on the line $x + y = 3$.
Putting $y = 0$, then $x = 3$.
This means that (3,0) is a point on the line $x + y = 3$.

To draw the line $x + y = 3$, join the points (0,3) and (3,0).

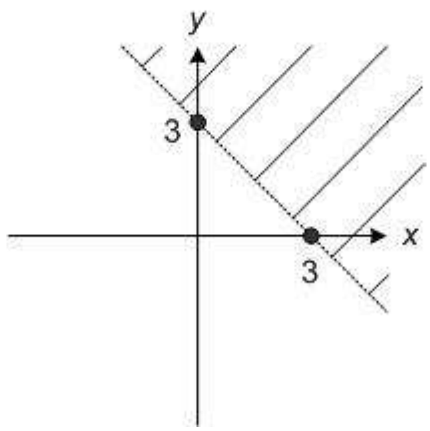


Fig. 4

$x + y > 3$ will consist of all points to the right of this line. Notice that the line is drawn as a dotted line to signify that the points on the line are not included in the region.

If you are unsure which side of the line (left or right) is the required region, proceed as follows:

Consider any point. The point (0,0) is often used as it results in easy calculations *i.e.* x=0, y=0.
The value of $x + y$ at this point is $0 + 0 = 0$, and 0 is not greater than 3.
So (0,0) does not give a value for which $x + y > 3$, and does not lie within the region. So the region will consist of all points on the opposite side of the line to (0,0).

We can now show the points which satisfy all 3 requirements, namely $x \geq 0$, $y < 0$, and $x + y > 3$ on the same diagram. The region includes points which lie on the line $y = 0$, but does **not** include points on the line $x + y = 3$.

We have already drawn all these regions in Figures 2, 3, and 4. Imagine all these figures lying on top of each other. Which region is shaded in all 3 figures? The answer is shown in Figure 5.

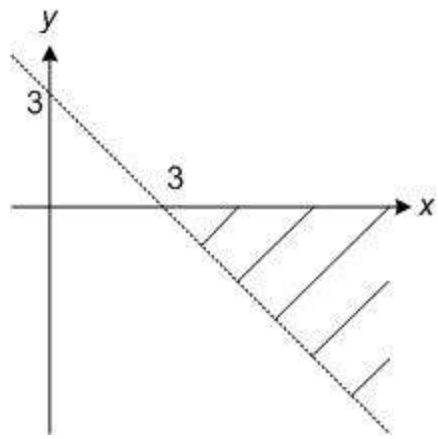


Fig. 5

Example 2

On separate diagrams shade the regions

a) $2 \le x \le 4$
b) $y \ge 2$
c) $x + y \le 5$

Then show the region $2 \le x \le 4$, $y \le 2$, $x + y \le 5$ on a single diagram.

Solution

a) $2 \le x \le 4$ means all points where x is between 2 and 4, including points on the line $x = 2$, and points on the line $x = 4$, as shown in Figure 6.

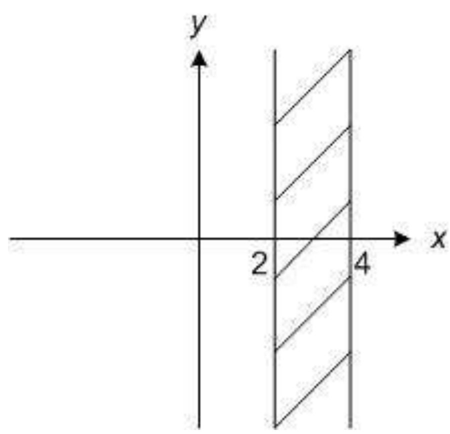


Fig. 6

b) $y \ge 2$ includes all points where y is greater than or equal to 2, so it includes all points on the line $y = 2$ as well as in the region above y=2, as shown in Figure 7.

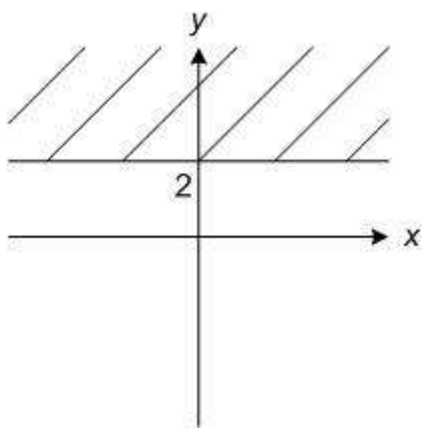


Fig. 7

c) $x + y \leq 5$ includes all points where $x + y$ is less than or equal to 5, so it includes all points on the line $x + y = 5$ as well as all points below this line, as shown in Figure 8.

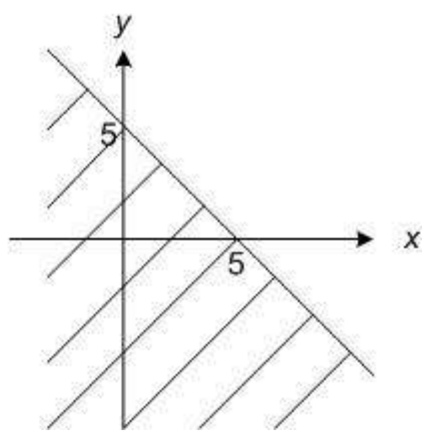


Fig. 8

The region where all 3 inequalities are satisfied is shown in Figure 9.

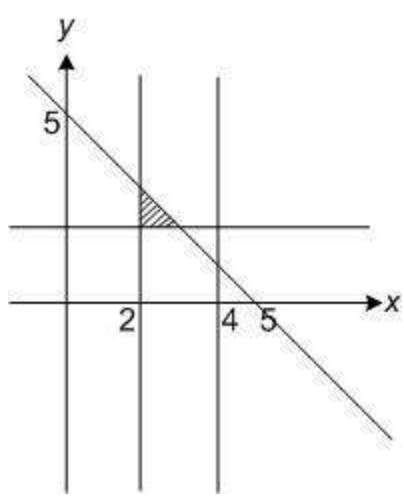


Fig. 9

Exercises

1. On separate diagrams shade the regions
   a) $x \leq 6$
   b) $y \geq -5$
   c) $x - y \geq 4$
   Then show the region where all 3 conditions are satisfied on a single diagram.

2. On separate diagrams shade the regions
   a) $x > -3$
   b) $y \leq 2$
   c) $x + y \leq 3$
   Then show the region where all 3 conditions are satisfied on a single diagram.

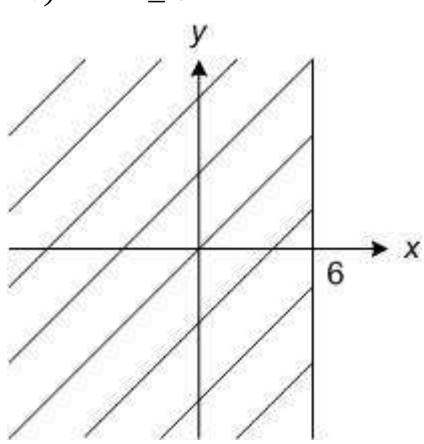
3. Show the region where all 3 of the following conditions are satisfied on a single diagram.
   a) $x \geq 0$
   b) $y \leq 1$
   c) $x + y \leq 2$

4. Show the region where all 3 of the following conditions are satisfied on a single diagram.
   a) $x \leq 0$
   b) $y \geq 0$
   c) $x + y > 2$

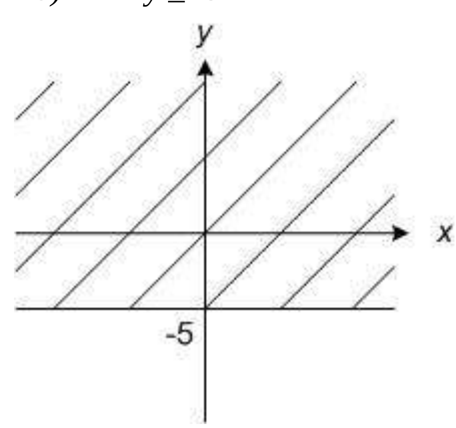
Solutions

1.

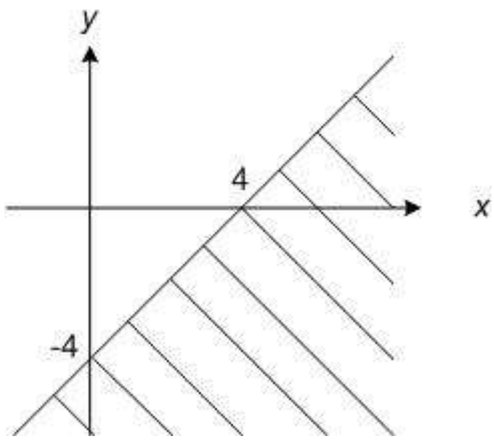
a) $x \leq 6$



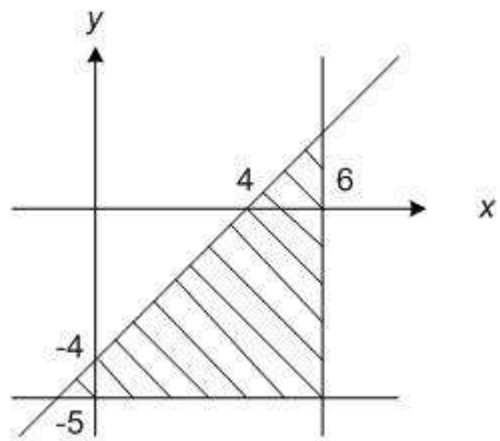
b) $y \geq -5$



c) $x - y \geq 4$

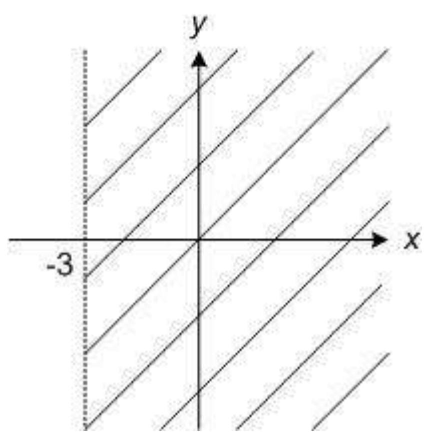


$x \leq 6, \quad y \geq -5, x - y \geq 4$

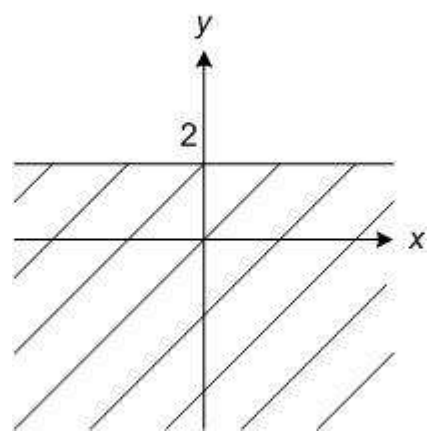


2.

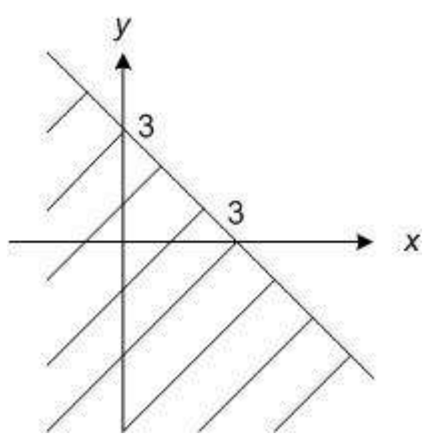
a) $x > -3$



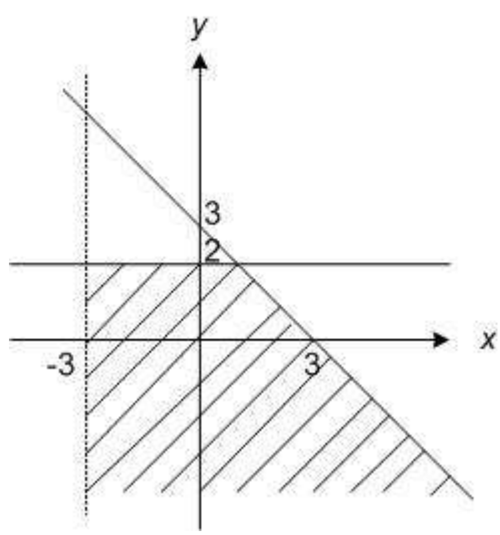
b) $y \leq 2$



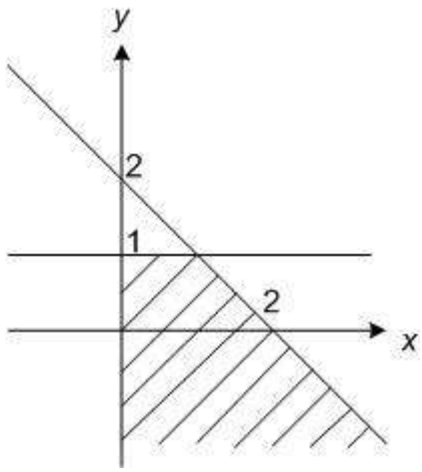
c) $x + y \leq 3$



$x > -3, y \leq 2, x + y \leq 3$



3. $x \geq 0, y \leq 1, x + y \leq 2$



4. $x \leq 0, y \geq 0, x + y > 2$