

Fractions, Decimals and Percentages

Objectives

On completion of this unit you should be able to:

1. Use a calculator to find a fraction.
2. Convert a fraction to a decimal fraction.
3. Find a percentage of a given number,
4. Increase or decrease an amount by a given percentage.
5. Express a quantity as a percentage or another.
6. Use the calculator fraction button.

Common fractions

Common fractions in everyday life are a half, quarter, third and so on. You often work these out automatically. For example, if you are told that the price of a £10 ticket has been reduced to half, you expect to pay £5 for it. You have divided 10 by 2 and obtained 5.

Now look at the following examples.

Example 1

Find one third of £24.

To find a third we need to divide £24 by 3.

$$£24 \div 3 = £8$$

One third of £24 is £8.

Example 2

Find $\frac{3}{4}$ of £24.

4

Find one quarter of £24 first.

Divide £24 by 4 to obtain £6.

This is one quarter of £24, we want to find three quarters so we need to multiply this answer by 3.

$$£6 \times 3 = £18$$

£18 is three quarters of £24.

Example 3

Find $\frac{4}{5}$ of £300.

5

We could use the same logic as before. First divide by 5 to find one fifth and then multiply the answer by four to find four fifths.

Alternatively we shall look at the way we calculate this question using a calculator.

It is useful to remember that 'times' can replace 'of' and so four fifths of £300 becomes:

$$\frac{4}{5} \times £300$$

Calculator: $4 \div 5 \times 300 = 240$

Four fifths of £300 is £240.

It is possible that you may have a fractions button on your calculator. If you do it will look like this:

$$a^{b/c}$$

Example 4

Using the fraction button on your calculator find

$$\frac{3}{8} \text{ of } £240.$$

Calculator: $3 \text{ } a^{b/c} \text{ } 8 \text{ } \times \text{ } 240 \text{ } = \text{ } 90$

Three eighths of £240 is £90.

Now try the following exercise.

Exercise A

Find

1. $\frac{1}{4}$ of £200

6. $\frac{1}{10}$ of £1000

2. $\frac{2}{5}$ of £500

7. $\frac{2}{3}$ of £6

3. $\frac{3}{4}$ of £700

8. $\frac{1}{9}$ of £72

4. $\frac{1}{8}$ of £320

9. $\frac{3}{4}$ of £100

5. $\frac{1}{3}$ of £960

10. $\frac{3}{10}$ of £90

Check your answers with those at the end of the booklet.

Decimals

Any fraction can be converted to a decimal fraction using your calculator. In fact you have already been doing this in the earlier exercise. Sometimes the result of a calculation is too long even to be shown completely on a calculator and so an approximate answer has to be given. Such an answer is given to a certain number of decimal places or significant figures. In the following examples we shall consider approximating to two decimal places where necessary.

Consider these examples.

Example 5

Convert $\frac{1}{2}$ to a decimal fraction.

Using your calculator:

Calculator: $1 \div 2 = 0.5$

Example 6

Convert $3\frac{1}{2}$ to a decimal fraction.

Calculator: $3 + 1 \div 2 = 3.5$

Example 7

Convert $\frac{5}{13}$ to a decimal fraction correct to two decimal places.

Calculator: $5 \div 13 = 0.3846153$

We consider the third figure after the decimal point. The number 4 is a number less than 5 so we ignore all following figures. (84 is closer to 80 than it is to 90).

The answer is $\frac{5}{13} = 0.38$ correct to two decimal places.

Example 8

Change $\frac{17}{7}$ to a decimal fraction correct to two decimal places.

Calculator: $17 \div 7 = 2.4285714$

Consider the third figure after the decimal point. This is the number 8. As 8 is greater than 5 we must add one on to the number before the 8. (28 is closer to 30 than to 20).

The answer is $\frac{17}{7} = 2.43$ correct to two decimal places.

Try the exercise on the next page.

Exercise B

Convert the following fractions to decimal fractions. Answer to 3 decimal places where necessary.

1. $\frac{2}{5}$

6. $\frac{70}{28}$

2. $\frac{3}{8}$

7. $\frac{18}{60}$

3. $\frac{25}{100}$

8. $2\frac{1}{4}$

4. $\frac{16}{50}$

9. $5\frac{1}{2}$

5. $\frac{65}{195}$

10. $16\frac{1}{4}$

Check your answers with those at the end of the unit.

Percentages

We often see percentages quoted in everyday life.

For example:

"Managers are to limit their salary increases to 4.5%."

"Sale. Reductions of 10%."

"VAT at 8% to be introduced on gas charges."

Study of this unit should help you to understand what is meant by these statements.

Finding a percentage

A percentage is a fraction of 100.

$\frac{20}{100}$ is the same as 20%.

Study the examples on the next page.

Example 9

Find 30% of £600.

Remember that 'of' means multiply.

We can rewrite this question

$$\frac{30}{100} \times £600 =$$

Calculator: $30 \div 100 \times 600 = 180$

30% of £600 is £180.

Example 10

Find 25% of £50.

Calculator: $25 \div 100 \times 50 = 12.5$

25% of £50 is £12.50. Although your calculator shows only 12.5, you must write £12.50 to show that it is £12 and 50p.

Complete the following exercise.

Exercise C

1. Find 20% of £200.
2. Find 50% of £1000.
3. Find 75% of £250.
4. Find 33% of £300.
5. Find 25% of £90.
6. Find 15% of £25.
7. Find 30% of £80.
8. Find 40% of £120.
9. Find 90% of £500.
10. Find 150% of £400.

Check your answers with those at the end of the booklet.

Increasing or decreasing an amount by a given percentage

Example 11

A table at present costing £500 is to be increased by 30%. What is the new price of the table?

First we need to find 30% of £500.

$$\frac{30}{100} \times £500 = £150$$

The cost of the table is to be **increased** by 30%.

30% of £500 is £150,

so the new cost is

$$£500 + £150 = £650.$$

Example 12

A sweater priced at £35 is to be reduced by 10% in a sale. What is the sale price?

Find 10% of £35.

$$\frac{10}{100} \times £35 = £3.50$$

The cost of the sweater is to be **decreased** by 10%.

10% of £35 is £3.50,

so the sale price is

$$£35 - £3.50 = £31.50.$$

Example 13

A bag priced at £19.99 is to be reduced in a sale. Find the sale price if the reduction is 10%.

10% of £19.99 is £1.999.

Sale price is

$$£19.99 - £1.999 = £17.991$$

This is £17.99 to the nearest penny.

Note: £19.991, £19.992, £19.993, £19.994
are all £19.99 to the nearest penny,
but,

£19.995, £19.996, £19.997, £19.998, £19.999
are all £20.00 to the nearest penny.

Now try the following exercise.

Exercise D

Increase the following prices by the given percentage.

- | | |
|-----------------|-------------------|
| 1. £2.30 by 20% | 6. £140 by 80% |
| 2. £3.40 by 50% | 7. £220 by 24% |
| 3. £24 by 10% | 8. £330 by 25% |
| 4. £320 by 15% | 9. £1000 by 75% |
| 5. £280 by 25% | 10. £2500 by 150% |

Decrease the following prices by the given percentage.

- | | |
|------------------|------------------|
| 11. £2.80 by 10% | 16. £1000 by 15% |
| 12. £3.40 by 15% | 17. £2500 by 80% |
| 13. £330 by 35% | 18. £24 by 6% |
| 14. £250 by 75% | 19. £68 by 30% |
| 15. £420 by 25% | 20. £55 by 82% |

Check your answers with those at the end of the booklet.

Expressing one quantity as a percentage of another

Suppose you are given a mark out of 10 in a test. It might be useful to quote this as a percentage.

Study these examples.

Example 14

You are given a mark of seven out of ten in a test. This could be written $\frac{7}{10}$. Change this mark to a percentage.

$\frac{7}{10}$ is a fraction.

To change a fraction or a decimal fraction to a percentage simply multiply by 100.

$$\frac{7}{10} \times 100 = 70\%.$$

Example 15

Forty out of two hundred BTEC students were over twenty five years old.
What percentage is this?

First express forty out of two hundred as a fraction,

$$\frac{40}{200}$$

then change the fraction to a percentage by multiplying by 100.

$$\frac{40}{200} \times 100 = 20$$

20% of BTEC students are over twenty five years old.

Example 16

Convert 0.12 to a percentage.

To change a fraction or a decimal fraction to a percentage simply multiply by 100.

Calculator: $0.12 \times 100 = 12$

0.12 is the same as 12%.

Try this exercise.

Exercise E

Express the following fractions or decimal fractions as percentages, correct to 1 decimal place where necessary.

1. $\frac{3}{4}$

6. 0.25

2. $\frac{4}{5}$

7. 0.43

3. $\frac{2}{3}$

8. 0.66

4. $\frac{1}{3}$

9. 1.25

5. $\frac{7}{8}$

10. 2.50

Check your answers with those at the end of the unit.

Use of the calculator fraction button

$a^{b/c}$

You have already used this button earlier in the unit. Now try using it when you study the following examples.

Using your calculator work through the following example.

Example 17

Using your calculator,

a) express $21\frac{35}{35}$ in its lowest terms,

b) find the value of

i) $100\frac{250}{250} + 1\frac{5}{5}$

ii) $10\frac{6}{6} \times 54\frac{40}{40}$

a) Calculator: $21 \text{ } a^{b/c} \text{ } 35 = 3 \text{ } \frac{35}{35}$

The answer is $3\frac{35}{35}$.

b)i) Calculator: $100 \text{ } a^{b/c} \text{ } 250 + 1 \text{ } a^{b/c} \text{ } 5 = 3 \text{ } \frac{35}{35}$

The answer is $3\frac{35}{35}$.

ii) Calculator: $10 \text{ } a^{b/c} \text{ } 6 \times 54 \text{ } a^{b/c} \text{ } 40 = 2 \text{ } \frac{1}{4}$

The answer is $2\frac{1}{4}$.

Try this short exercise using your calculator.

Exercise F

Express the fractions in 1 to 4 in their lowest terms.

1. $\frac{30}{90}$

4. $\frac{45}{63}$

2. $\frac{120}{160}$

5. $\frac{45}{20} + \frac{21}{60}$

3. $\frac{20}{60}$

6. $\frac{80}{50} \div \frac{23}{150}$

Check your answers before going onto the miscellaneous exercise on the next page. Remember to use your calculator to cancel any fractions to their lowest terms where necessary.

Exercise G

1. Jane has £8 to spend. She spends £2 of it on cosmetics. What percentage of the £8 does she spend on cosmetics?
2. A farmer has 135 cows, 27 goats and 38 pigs.
 - a) What is the total number of animals?
 - b) What fraction of the animals are goats?
 - c) What percentage of the animals are goats?
3. A box of chocolates weighs 900g. Included in this is 45g. of packaging.
 - a) Express the weight of the packaging as a fraction of the total weight. Express this fraction in its lowest terms.
 - b) Express the fraction in a) as a percentage.
4. A shopkeeper buys 10 irons at £12 each. He sells 4 of them at £20 each. He then reduces the price of the remaining stock by 15%.
 - a) What is the cost price to the shopkeeper of 10 irons at £12 each?
 - b) How much does he obtain from the sale of the first 4 irons?
 - c) What is the reduced price of each of the next 6 irons?
 - d) Find the total sale price of all 10 irons.
 - e) Express the cost price of the 10 irons as a fraction of the total sale price. Express the fraction in its lowest terms.
 - f) Express the cost price of the 10 irons as a percentage of the total sale price. Answer to one decimal place.
5. Carol pays 8% of her salary into a savings account. If her salary is £250 per month,
 - a) How much does she pay into the savings account each month?
 - b) Express the amount she pays each month as a fraction of her monthly salary, giving your answer in its lowest terms.
6. A shop sells a 3 hour video tape for £3.99. When John goes to the shop he sees a notice which says 'Buy 7 for only £22'.
 - a) How much will he save if he buys 7 for £22 rather than 7 at £3.99 each?
 - b) What percentage discount is it, based on the price of 7 videos at £3.99 each? Answer correct to two decimal places.

Exercise G continued on the next page.

Exercise G (Continued)

7. While out shopping, Mrs. Taylor notices that the cereal she means to buy is sold in two sizes, a standard size of 200g. costing £1.10 and a family size of 300g. costing £1.50.
 - a) Find the saving she would make if she bought 2 family size packs instead of 3 standard size.
 - b) Express this saving as a percentage of the cost of 3 standard size packs. Give your answer correct to two decimal places.
8. In his end of year examinations, Jason scored 67 out of 100 in Mathematics, 57 out of 80 in English and 42 out of 60 in French. Find each of his marks as a percentage and determine which subject shows the highest standard of work.
9. Simon wishes to buy a laptop computer and he can either pay £990 cash or £1150 on credit.
 - a) Find the amount saved if the computer is bought for cash.
 - b) Express this saving as a percentage of the cash price, giving your answer correct to 2 decimal places.
10. The tickets for a football match are advertised as costing £1.50 standing or £5.50 for a seat, if they are ordered in advance. All ticket prices will be increased by 5% if purchased on the day of the match. Find the price of each type of ticket bought on the day of the match. Answer correct to the nearest penny.

Check your answers with those at the end of the booklet. Ask your tutor for help if you experienced any difficulties.

Answers

Exercise A

1. £50
2. £200
3. £525
4. £40
5. £320
6. £100
7. £4
8. £8
9. £75
10. £27

Exercise B

1. 0.4
2. 0.375
3. 0.25
4. 0.32
5. 0.33333333 This is written as 0.333 correct to 3 decimal places.
6. 2.5
7. 0.3
8. 2.25
9. 5.5
10. 16.25

Exercise C

1. £40
2. £500
3. £187.50
4. £99
5. £22.50
6. £3.75
7. £24
8. £48
9. £450
10. £600

Exercise D

- | | |
|------------|-------------|
| 1. £2.76 | 11. £2.52 |
| 2. £5.10 | 12. £2.89 |
| 3. £26.40 | 13. £214.50 |
| 4. £368 | 14. £62.50 |
| 5. £350 | 15. £315 |
| 6. £252 | 16. £850 |
| 7. £272.80 | 17. £500 |
| 8. £412.50 | 18. £22.56 |
| 9. £1750 | 19. £47.60 |
| 10. £6250 | 20. £9.90 |

Exercise E

1. 75%
2. 80%
3. 66.666667% This is 66.7% correct to one decimal place.
4. 33.333333% This is 33.3% correct to one decimal place.
5. 87.5%
6. 25%
7. 43%
8. 66%
9. 125%
10. 250%

Exercise F

- | | |
|------------------|----------------------|
| 1. $\frac{1}{3}$ | 4. $\frac{5}{7}$ |
| 2. $\frac{3}{4}$ | 5. $2\frac{3}{5}$ |
| 3. $\frac{1}{3}$ | 6. $10\frac{10}{23}$ |

Exercise G

1. 25%
2. a) 200 b) $\frac{27}{200}$
3. a) $\frac{45}{900} = \frac{1}{20}$
4. a) £120 b) £80
5. a) £20 c) £17 d) £182
6. a) £5.93 e) $\frac{120}{182} = \frac{60}{91}$
7. a) 30p f) 65.9% correct to one decimal place.
8. Maths 67%, English 71.25%, French 70%, English showed the highest standard.
9. a) £160 b) 21.23% correct to two decimal places.
10. Standing £1.58, Seat £5.78